Asymptotic Notation Worksheet[1]

Name:____

Provide a clear, complete, and concise answer to each question. If a problem asks for a proof, you are to provide as rigorous of a proof as possible.

1. Give a Θ bound for each of the following functions. You do not need to prove it.

   (a) $f(x) = x^5 + x^3 + 1900 + x^7 + 21x + x^2$

   (b) $f(x) = (x^2 + 23x + 19)(x^2 + 23x + x^3 + 19)x^3$ (Don't make this one harder than it is)

   (c) $f(x) = x^2 + 10,000x + 100,000,000,000$

   (d) $f(x) = 49 * 2^x + 34 * 3^x$

   (e) $f(x) = 2^x + x^5 + x^3$

   (f) $f(x) = xlogx + x^2$

   (g) $f(x) = log^{300}x + x^{.000001}$

2. (2.1 #9, p52) Indicate whether the first function of each of the following pairs has a smaller, same, or larger order of growth (to within a constant multiple) than the second function. Use Θ, Ω, or *Big-O* notation to express the relationship. You do not need to prove it.

   (a) $n(n+1)$ vs $2000n^2$

   (b) $100n^2$ vs $.01n^3$

   (c) $\log_2 n$ vs $\ln n$

   (d) $\log_2^2 n$ vs $\log_2 n^2$

   (e) 2^{n-1} vs 2^n

   (f) $(n-1)!$ vs $n!$

[1]Some problems are taken and/or modified from *An Introduction to the Design and Analysis of Algorithms* by Anany Levitin. This is indicated in parentheses after the problem number (Section, problem, page).

3. Determine whether each of the following statements is true or false.

Question	T/F?
If $f(x) = O(g(x))$, then $f(x)$ grows faster than $g(x)$	
If $f(x) = \Theta(g(x))$, then $f(x)$ grows faster than $g(x)$	
If $f(x) = O(g(x))$, then $f(x)$ grows at the same rate as $g(x)$	
If $f(x) = \Omega(g(x))$, then $f(x)$ grows faster than $g(x)$	
If $f(x) = O(g(x))$, then $f(x) = \Omega(g(x))$	
If $f(x) = \Theta(g(x))$, then $f(x) = O(g(x))$	
If $f(x) = O(g(x))$, then $f(x) = \Theta(g(x))$	
If $f(x) = O(g(x))$, then $g(x) = O(f(x))$	
If $f(x) = O(g(x))$, then $g(x) = \Omega(f(x))$	
If $f(x) = \Theta(g(x))$, then $f(x) = \Omega(g(x))$ and $f(x) = O(g(x))$	
If $f_1(x) = O(g_1(x))$ and $f_2(x) = O(g_2(x))$, then $f_1(x) + f_2(x) = O(max(g_1(x), g_2(x)))$	
$f(x) = O(g(x))$ iff $f(x) = \Theta(g(x))$	
$f(x) = O(g(x))$ iff $g(x) = O(f(x))$	
$f(x) = O(g(x))$ iff $g(x) = \Omega(f(x))$	
$f(x) = \Theta(g(x))$ iff $f(x) = \Omega(g(x))$ and $f(x) = O(g(x))$	
If $f(x) = O(g(x))$ and $g(x) = O(h(x))$, then $f(x) = O(h(x))$	

4. (2.2 #2, p60) Use *LIMITS* to prove or disprove each of the following statements. Show your work.

   (a) $n(n+1)/2 \in O(n^3)$

   (b) $n(n+1)/2 \in O(n^2)$

   (c) $n(n+1)/2 \in \Theta(n^3)$

   (d) $n(n+1)/2 \in \Omega(n)$

5. (2.2 #3, p60) For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. Use the simplest $g(n)$ possible in your answers. Prove your assertions using the either the definition of Θ or using limits, but you must use each technique at least once.

   (a) $(n^2+1)^{10}$

   (b) $\sqrt{10n^2+7n+3}$

   (c) $2n\log((n+2)^2)+(n+2)^2\log\frac{n}{2}$

   (d) $2^{n+1}+3^{n-1}$

6. Let α and β be positive real numbers such that $\alpha < \beta$.

   (a) Prove that $n^\alpha \in O(n^\beta)$, but that $n^\beta \notin \Theta(n^\alpha)$

   (b) Prove that $\alpha^n \in O(\beta^n)$, but that $\beta^n \notin \Theta(\alpha^n)$

   (c) Prove that $\log_\alpha n \in \Theta(\log_\beta n)$

7. For each of the following, determine whether $f(n) \in O(g(n))$, $f(n) \in \Omega(g(n))$, or both ($f(n) \in \Theta(g(n))$). Prove your answer.

   (a) $f(n) = \sqrt{n}, g(n) = \sqrt[3]{n}$

   (b) $f(n) = \frac{n^3}{25} + \log(n^2), g(n) = n^4$