

## Best-First Search with branch-and-bound pruning example: 0-1 Knapsack

The same problem from above (with  $W=9$ )

| $i$ | $p_i$ | $w_i$ | $p_i/w_i$ |
|-----|-------|-------|-----------|
| 1   | \$10  | 2     | 5         |
| 2   | \$15  | 5     | 3         |
| 3   | \$10  | 4     | 2.5       |
| 4   | \$12  | 5     | 2.4       |
| 5   | \$6   | 3     | 2         |
| 6   | \$4   | 4     | 1         |

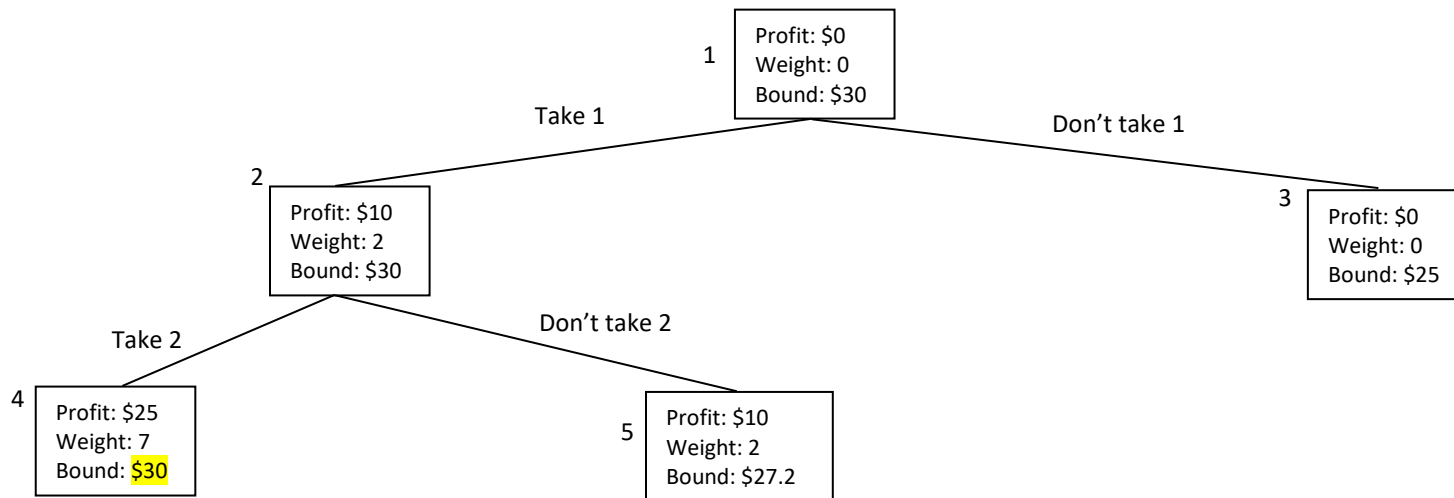
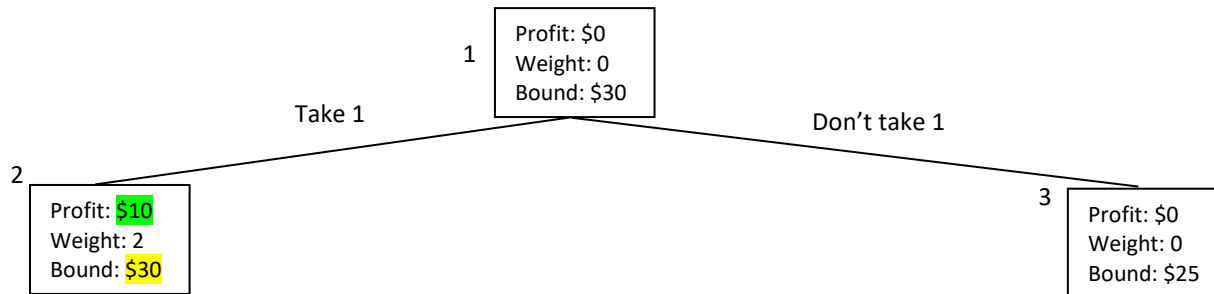
|    |                                  | Take? | Profit | weight | bound | Max profit | Action    | Priority Queue                       |
|----|----------------------------------|-------|--------|--------|-------|------------|-----------|--------------------------------------|
| 1  | Root node                        |       | 0      | 0      | 30    | 0          | Insert 1  | 1 (30)                               |
| 2  | Remove 1: 1st child              | 1     | 10     | 2      | 30    | 10         | Insert 2  | 2 (30)                               |
| 3  | 2nd child of 1                   | not 1 | 0      | 0      | 25    | 10         | Insert 3  | 2 (30), 3 (25)                       |
| 4  | Remove 2: 1 <sup>st</sup> child  | 2     | 25     | 7      | 30    | 25         | Insert 4  | 3 (25), 4 (30)                       |
| 5  | 2nd child of 2                   | not 2 | 10     | 2      | 27.2  | 25         | Insert 5  | 3 (25), 5 (27.2), 4 (30)             |
| 6  | Remove 4: 1st child              | 3     | 35     | 11     | 0     | 25         |           | 3 (25), 5 (27.2)                     |
| 7  | 2nd child of 4                   | not 3 | 25     | 7      | 29.8  | 25         | Insert 7  | 3 (25), 5 (27.2), 7 (29.8)           |
| 8  | Remove 7: 1st child              | 4     | 37     | 12     | 0     | 25         |           | 3 (25), 5 (27.2)                     |
| 9  | 2 <sup>nd</sup> child of 7       | Not 4 | 25     | 7      | 29    | 25         | Insert 9  | 3 (25), 5 (27.2), 9 (29)             |
| 10 | Remove 9: 1st child              | 5     | 31     | 10     | 0     | 25         |           | 3 (25), 5 (27.2)                     |
| 11 | 2 <sup>nd</sup> child of 9       | Not 5 | 25     | 7      | 27    | 25         | Insert 11 | 3 (25), 11 (27), 5 (27.2),           |
| 12 | Remove 5: 1st child              | 3     | 20     | 6      | 27.2  | 25         | Insert 12 | 3 (25), 11 (27), 12 (27.2)           |
| 13 | 2nd child of 5                   | Not 3 | 10     | 2      | 26    | 25         | Insert 13 | 3 (25), 13 (26), 11 (27)             |
| 14 | Remove 12: 1st child             | 4     | 32     | 11     | 0     | 25         |           | 3 (25), 13 (26), 11 (27)             |
| 15 | 2 <sup>nd</sup> child of 12      | Not 4 | 20     | 6      | 26    | 25         | Insert 15 | 3 (25), 13 (26), 15 (26),<br>11 (27) |
| 16 | Remove 11: 1 <sup>st</sup> child | 6     | 29     | 11     | 0     | 26         |           | 3 (25), 13 (26), 15 (26)             |
| 17 | 2 <sup>nd</sup> child of 11      | Not 6 | 25     | 7      | 25    | 26         |           | 3 (25), 13 (26)                      |
| 18 | Remove 15: 1st child             | 5     | 26     | 9      | 26    | 26         |           | 3 (25), 13 (26)                      |
| 19 | 2 <sup>nd</sup> child of 15      | Not 5 | 20     | 6      | 23    | 26         |           | 3 (25), 13 (26)                      |
|    | Remove 13                        |       |        |        |       |            |           | 3 (25)                               |
|    | Remove 3                         |       |        |        |       |            |           |                                      |

In the last 2 steps we remove the nodes and do nothing since they are no longer promising

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| i | p <sub>i</sub> | w <sub>i</sub> | p <sub>i</sub> /w <sub>i</sub> |
|---|----------------|----------------|--------------------------------|
| 1 | 10             | 2              | 5                              |
| 2 | 15             | 5              | 3                              |
| 3 | 10             | 4              | 2.5                            |
| 4 | 12             | 5              | 2.4                            |
| 5 | 6              | 3              | 2                              |
| 6 | 4              | 4              | 1                              |

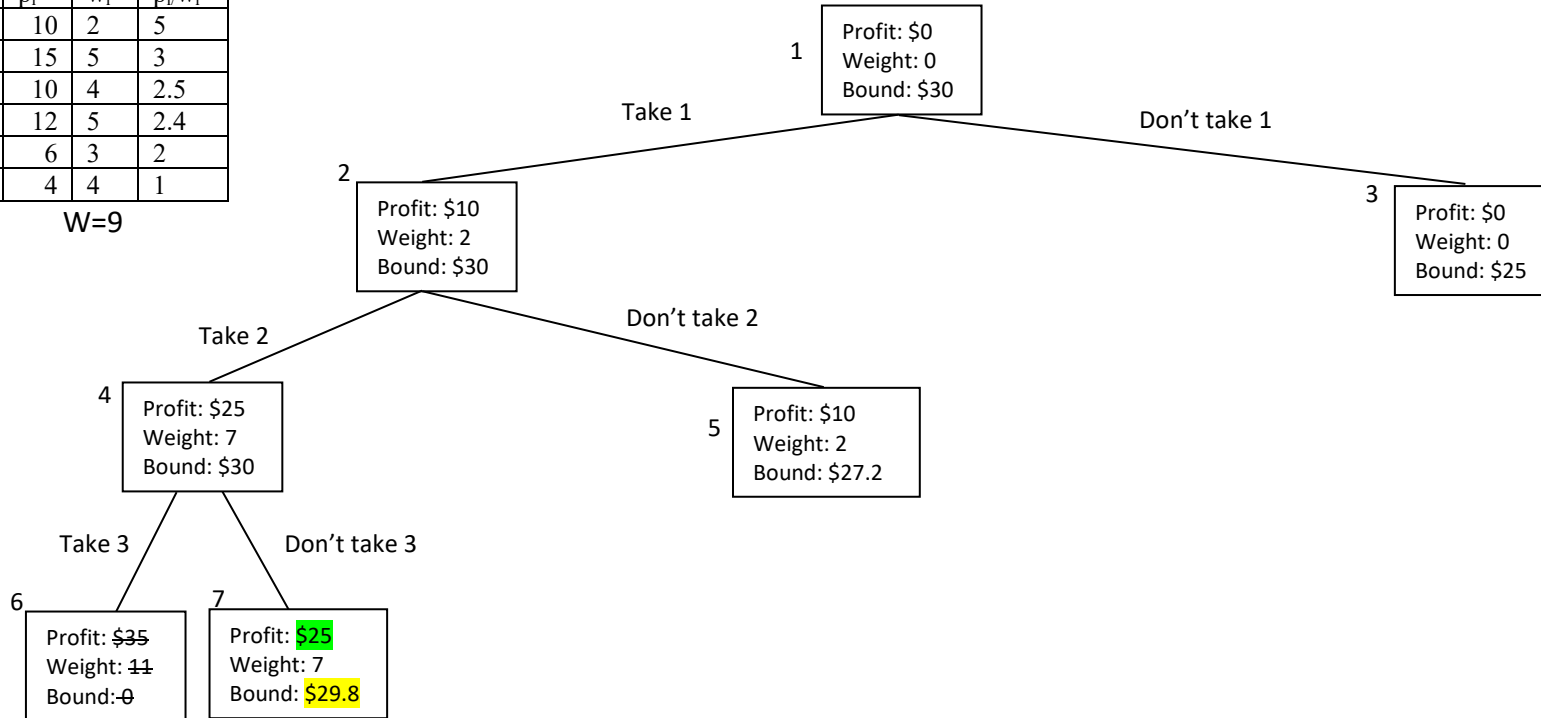
W=9



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| 3   | 10    | 4     | 2.5       |
| 4   | 12    | 5     | 2.4       |
| 5   | 6     | 3     | 2         |
| 6   | 4     | 4     | 1         |

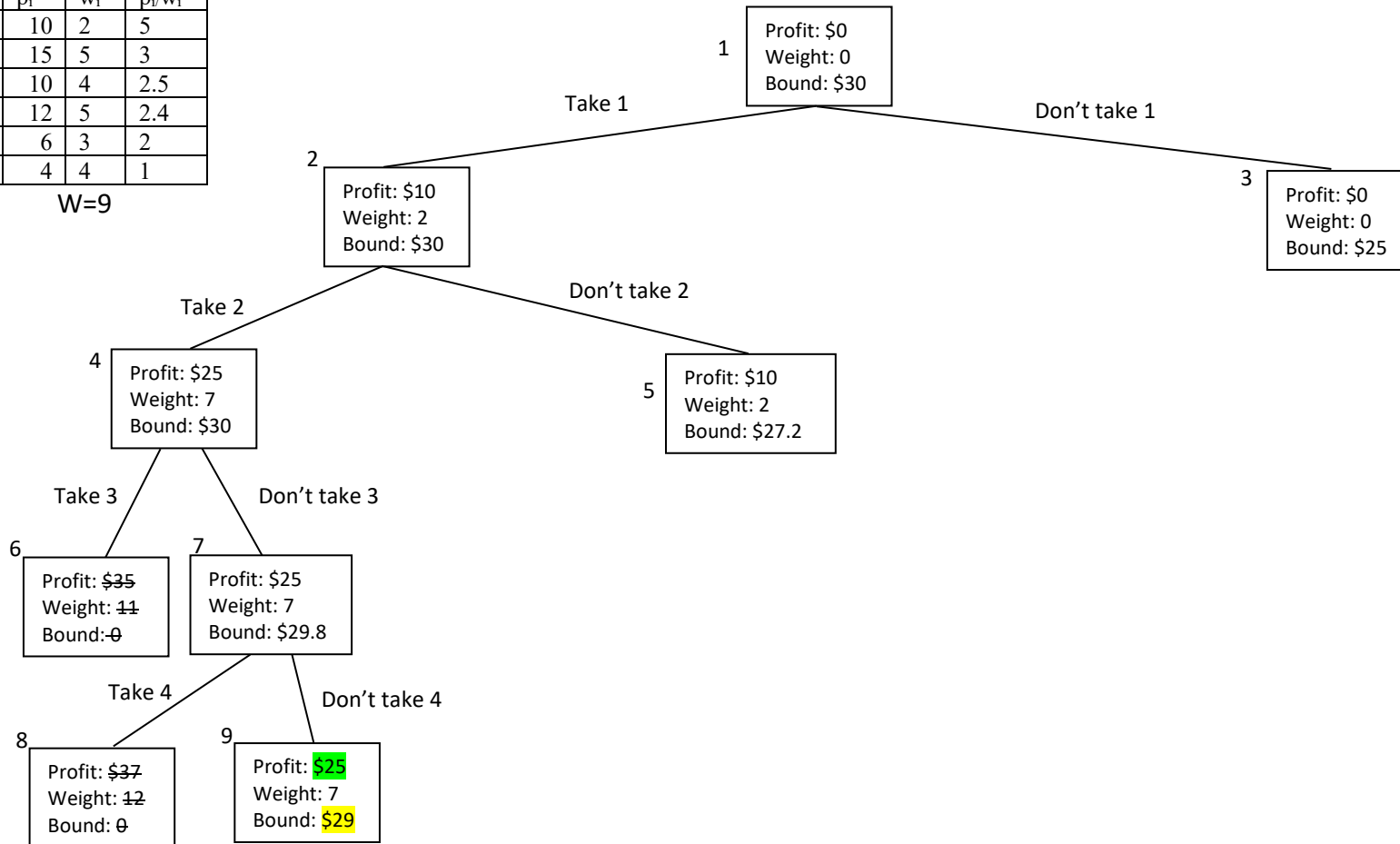
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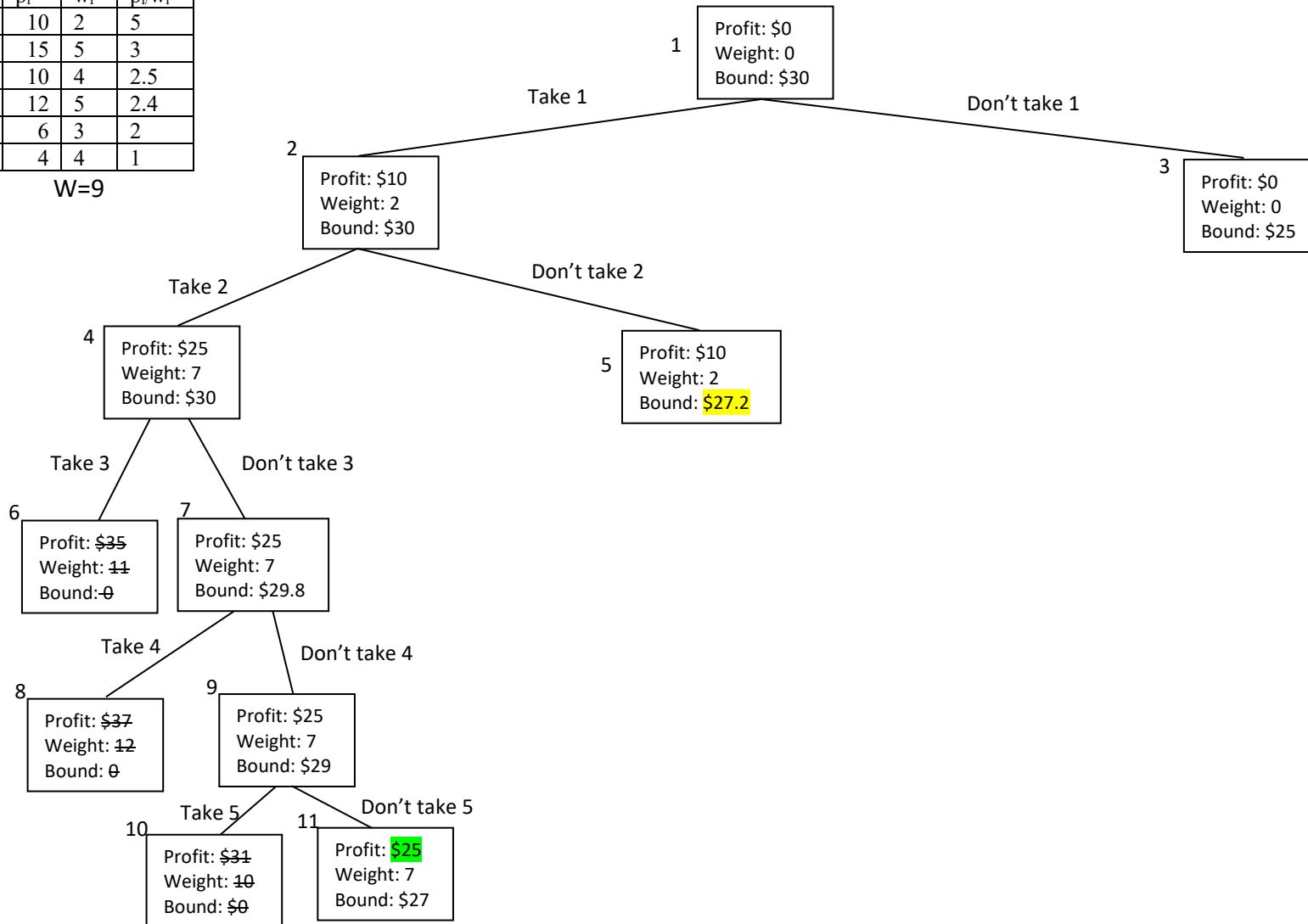
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| i | p <sub>i</sub> | w <sub>i</sub> | p <sub>i</sub> /w <sub>i</sub> |
|---|----------------|----------------|--------------------------------|
| 1 | 10             | 2              | 5                              |
| 2 | 15             | 5              | 3                              |
| 3 | 10             | 4              | 2.5                            |
| 4 | 12             | 5              | 2.4                            |
| 5 | 6              | 3              | 2                              |
| 6 | 4              | 4              | 1                              |

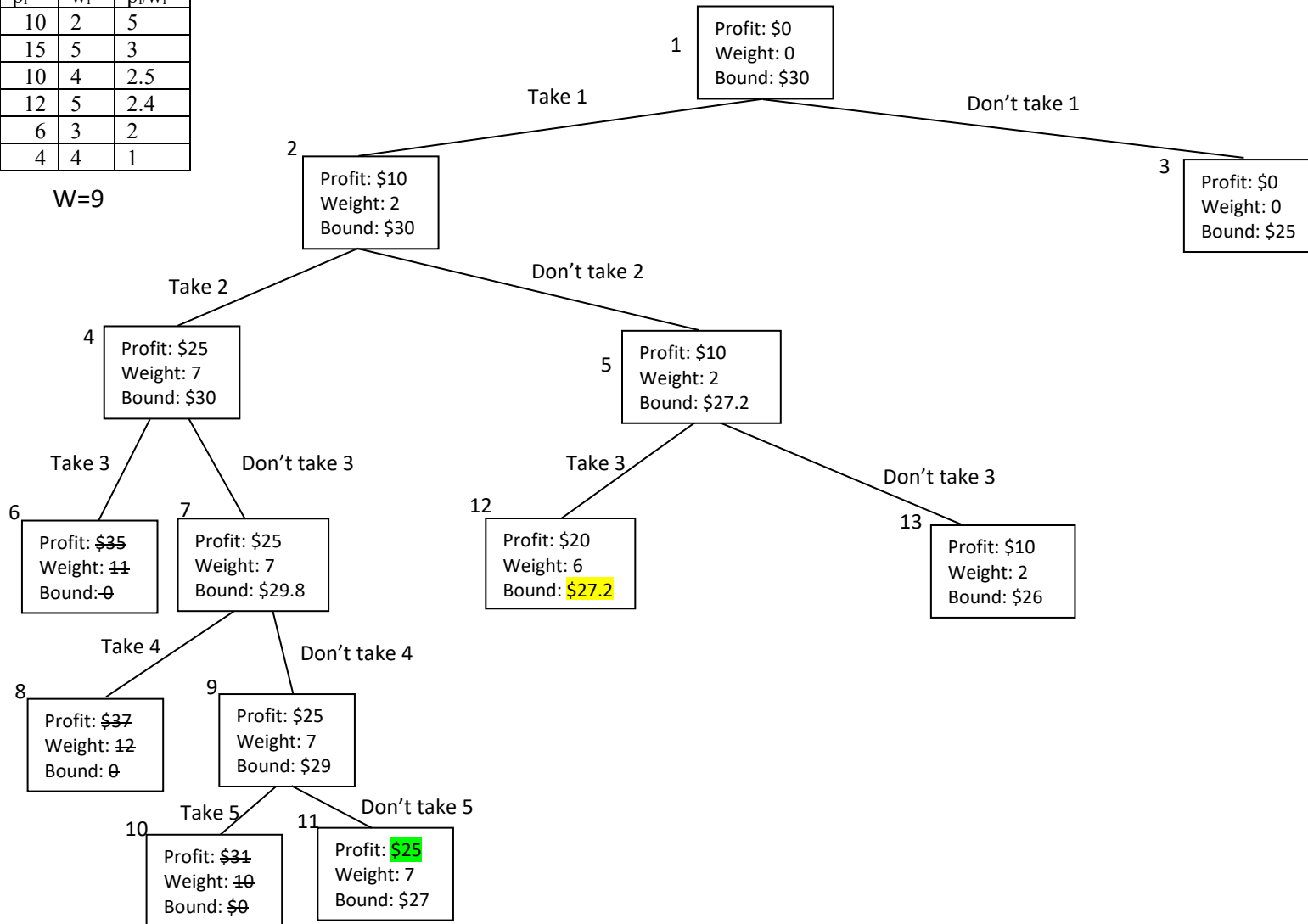
W=9



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| i | p <sub>i</sub> | w <sub>i</sub> | p <sub>i</sub> /w <sub>i</sub> |
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| 4 | 12             | 5              | 2.4                            |
| 5 | 6              | 3              | 2                              |
| 6 | 4              | 4              | 1                              |

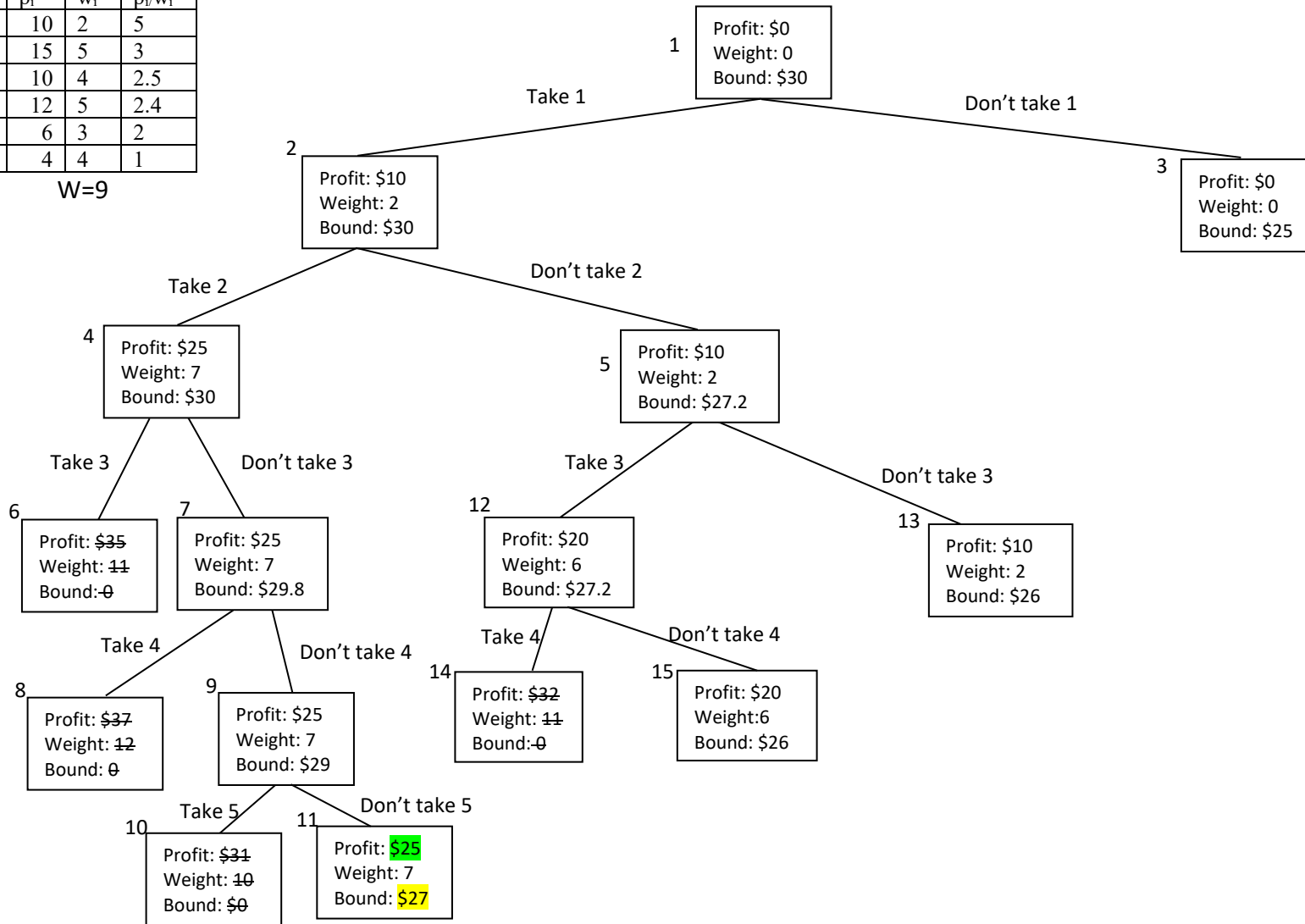
W=9



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| i | p <sub>i</sub> | w <sub>i</sub> | p <sub>i</sub> /w <sub>i</sub> |
|---|----------------|----------------|--------------------------------|
| 1 | 10             | 2              | 5                              |
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| 5 | 6              | 3              | 2                              |
| 6 | 4              | 4              | 1                              |

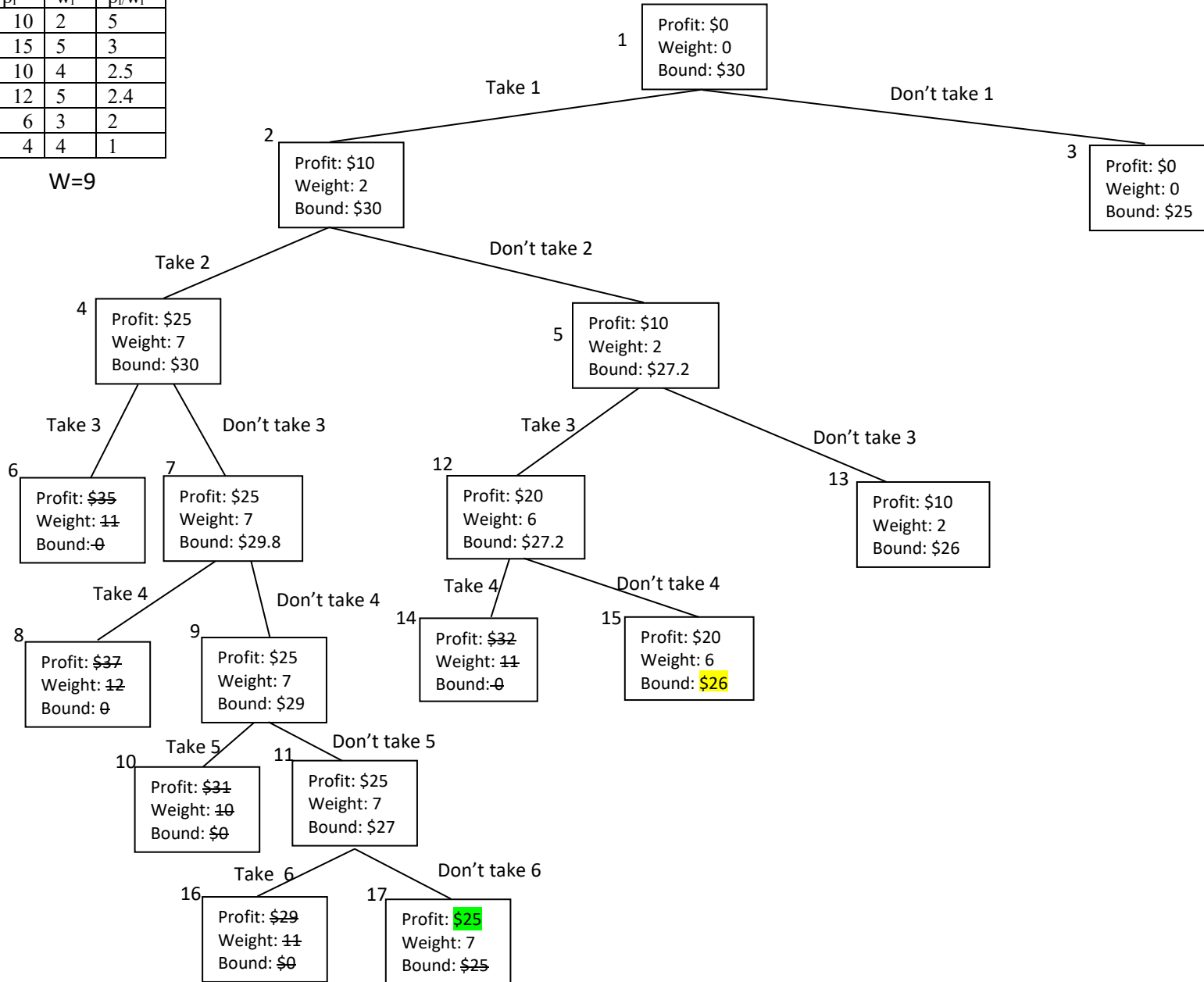
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| 3 | 10             | 4              | 2.5                            |
| 4 | 12             | 5              | 2.4                            |
| 5 | 6              | 3              | 2                              |
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W=9

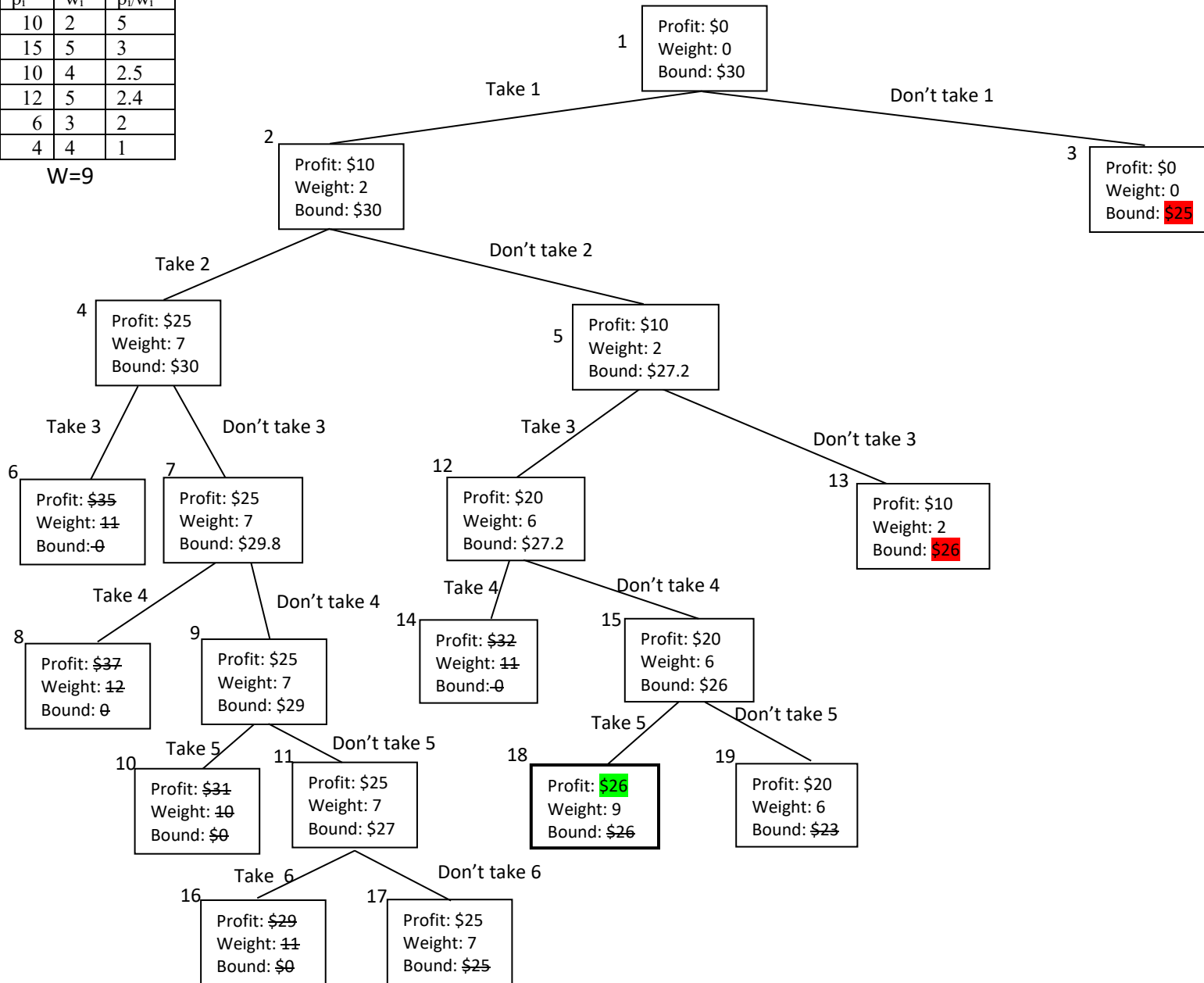




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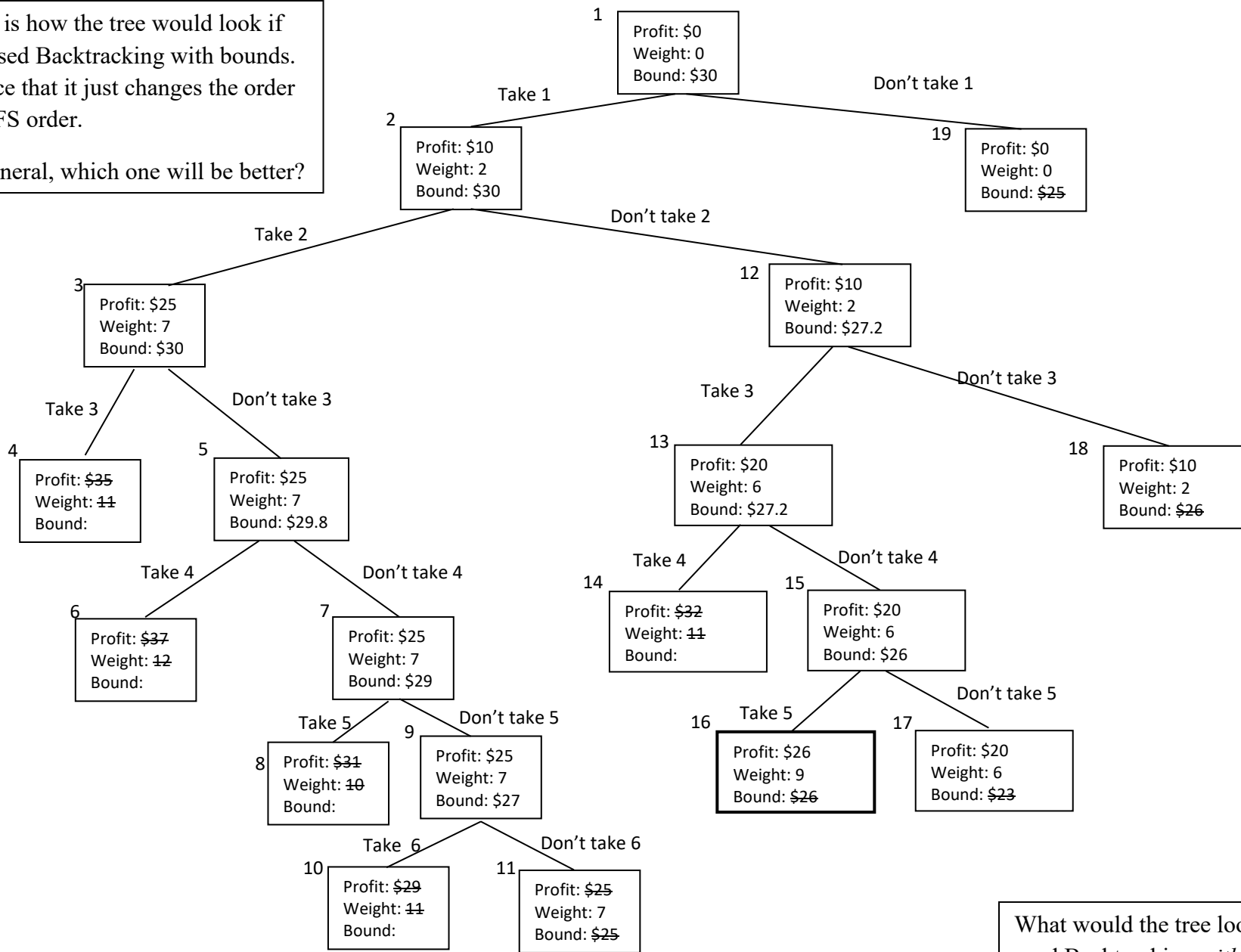
| $i$ | $p_i$ | $w_i$ | $p_i/w_i$ |
|-----|-------|-------|-----------|
| 1   | 10    | 2     | 5         |
| 2   | 15    | 5     | 3         |
| 3   | 10    | 4     | 2.5       |
| 4   | 12    | 5     | 2.4       |
| 5   | 6     | 3     | 2         |
| 6   | 4     | 4     | 1         |

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## Best-First Search with branch-and-bound pruning example: 0-1 Knapsack

Here is how the tree would look if we used Backtracking with bounds. Notice that it just changes the order to DFS order. In general, which one will be better?



What would the tree look like if we used Backtracking *without* bounds?