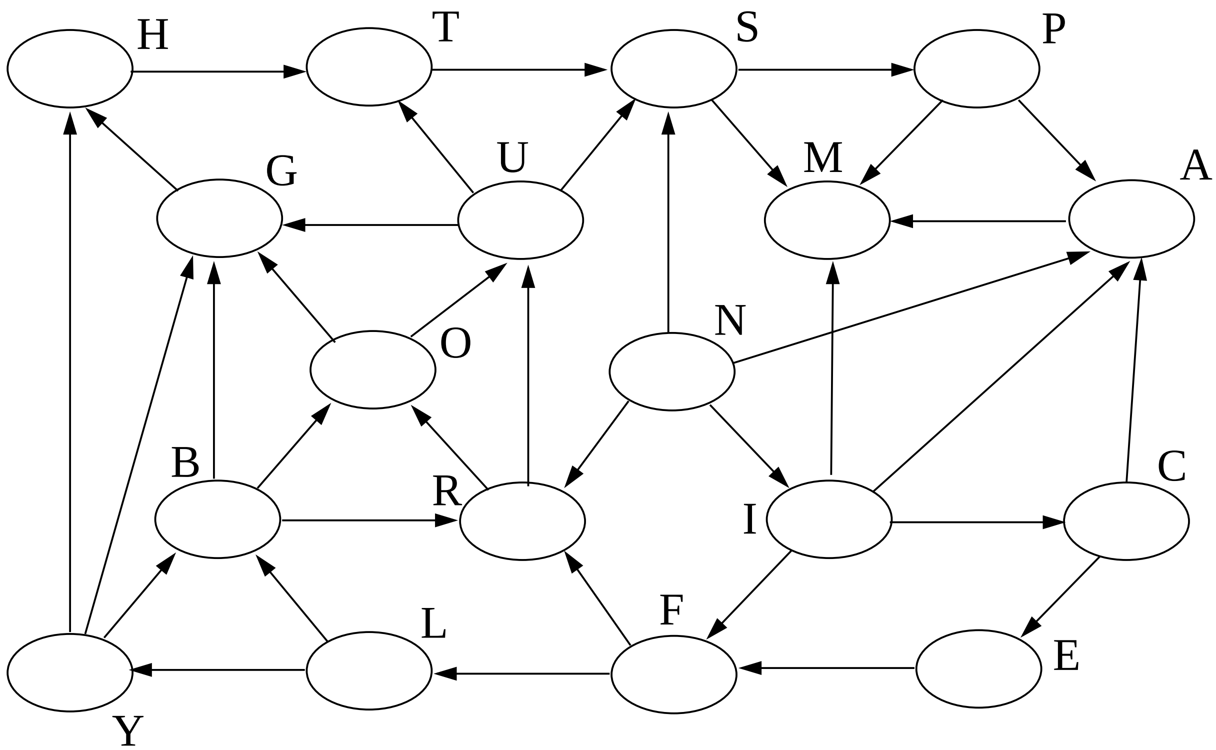
Name:____

1. The following graph represents a series of tasks that I must perform. The arrows represent prerequisites. For instance, I cannot do task **H** until both tasks **G** and **Y** are complete.



(a) Use DFS to determine a valid order in which I can complete the tasks. Assume adjacency lists are stored in alphabetical order.

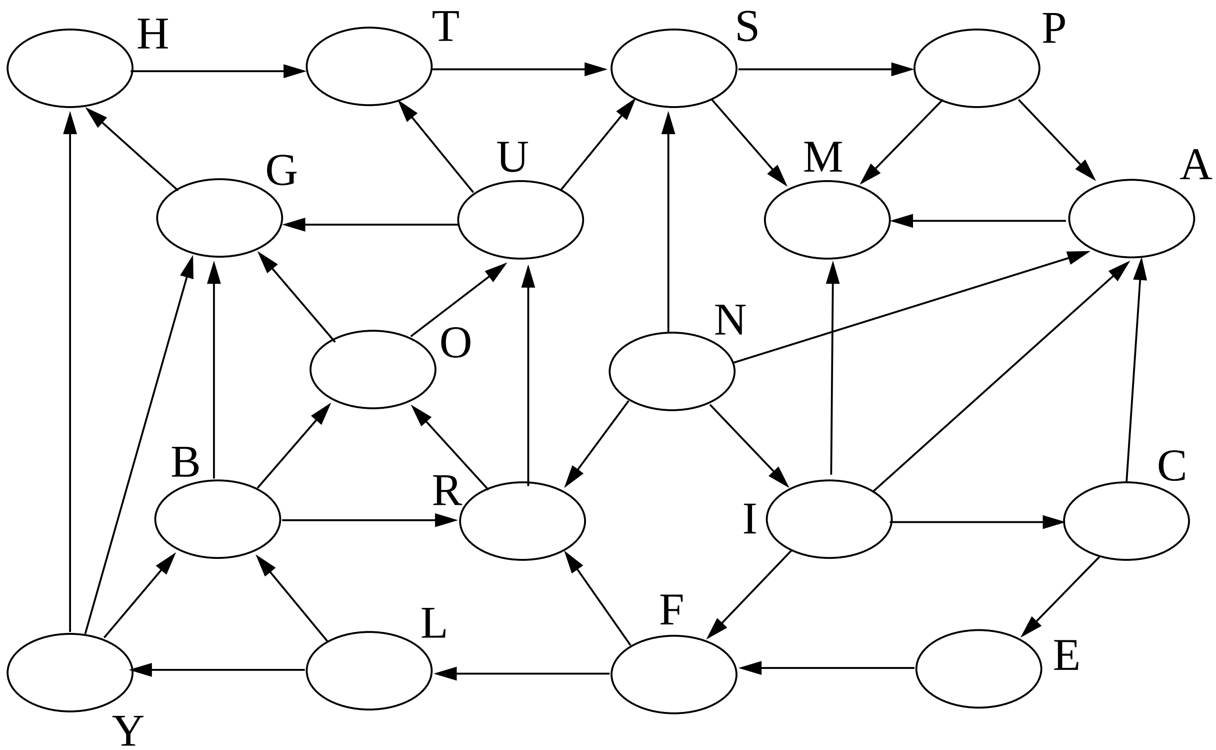
- Show the timestamps in the vertices.
- Draw the DFS tree edges on the graph, numbering them in the order they were added.
- Give the valid order to complete the tasks here.

(b) Does the graph contain a *mother* vertex? If so, which vertex? If not, why not?

(c) Describe a decrease-and-conquer algorithm to solve this problem.

2. Determine the shortest path from **F** to every other reachable vertex in the following graph. Assume adjacency lists are stored in alphabetical order.

   - Show the distance from **F** to each vertex in the vertex. Leave them blank if a vertex cannot be reached.
   - Draw the edges of the BFS tree on the graph, numbering them in the order they were added.



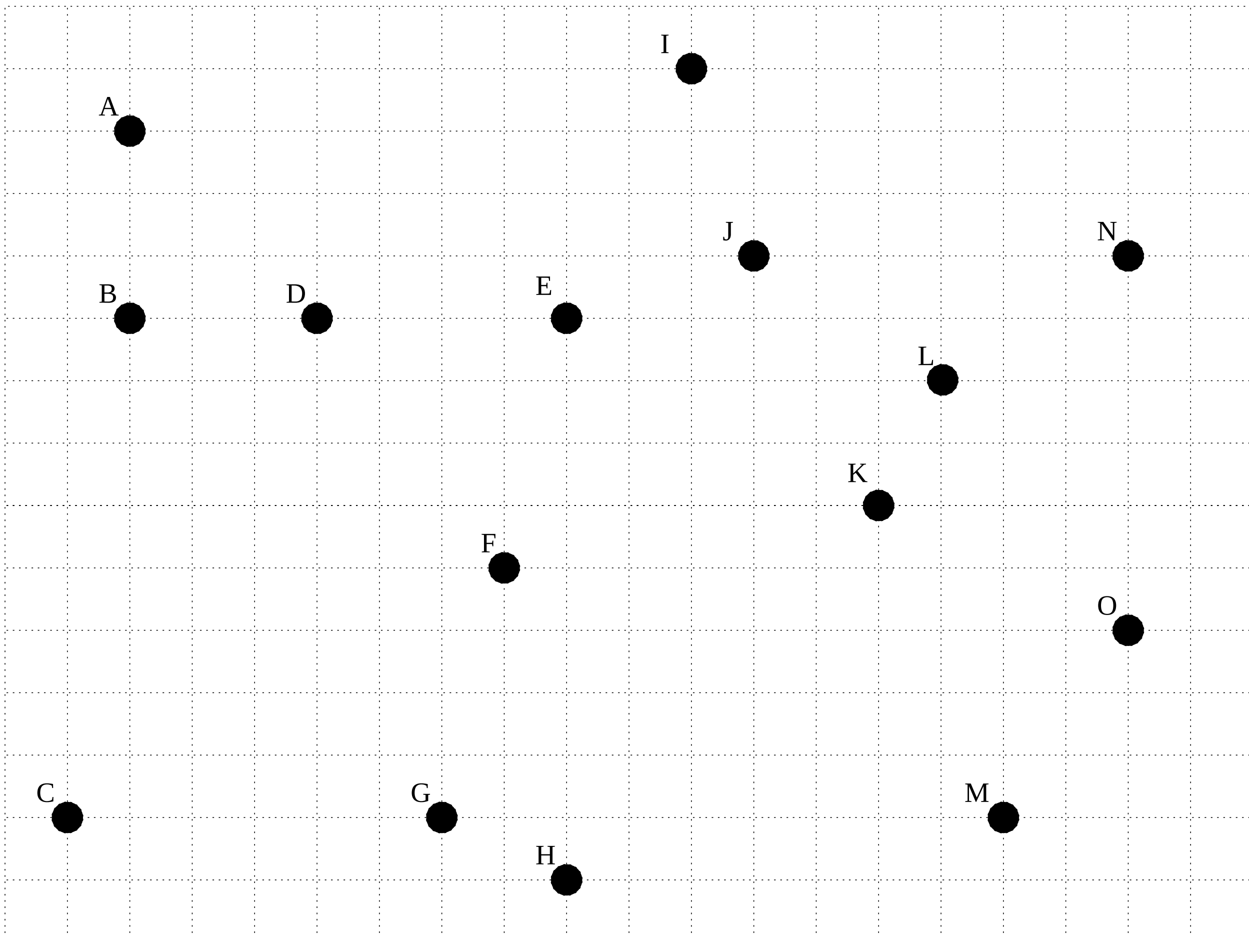
3. If a directed graph can be topologically sorted, does it have a mother vertex? Explain or give a counterexample.

4. If a directed graph has a mother vertex, can it can be topologically sorted? Explain or give a counterexample.

5. Is it true that a directed graph has at least one strongly connected component consisting of more than one vertex if and only if there is no valid topological sort. Explain or give a counterexample.

6. An Internet provider is installing an optical network in a rural town. The map below indicates the locations of the houses and the Internet provider's office (A). Each grid line represents 1 mile. They can route the cables anywhere, can split at any house as needed (but only split at houses), and only need one connection to each house. Since optical cable is really expensive, the goal is to minimize the amount of cable required.
Note: If they want to route from **I** to **J**, they could route south 3 miles, and east 1 mile for a total of 4 miles, but it would be better to route in a straight line from **I** to **J** for a cost of $\sqrt{3^2+1^2}=\sqrt{10}\approx 3.2$ miles.

   (a) Use Kruskal's algorithm to determine where they should put the wires.
      - Draw the wires on the map below.
      - For each wire give
         - its length
         - the order it was selected (in parentheses after the length)
      - Give the total length of the wires.



$\sqrt{2}\approx 1.4$
$\sqrt{5}\approx 2.2$
$\sqrt{8}\approx 2.8$
$\sqrt{10}\approx 3.2$
$\sqrt{13}\approx 3.6$
$\sqrt{17}\approx 4.1$
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$\sqrt{20}\approx 4.5$
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$\sqrt{29}\approx 5.4$
$\sqrt{32}\approx 5.7$
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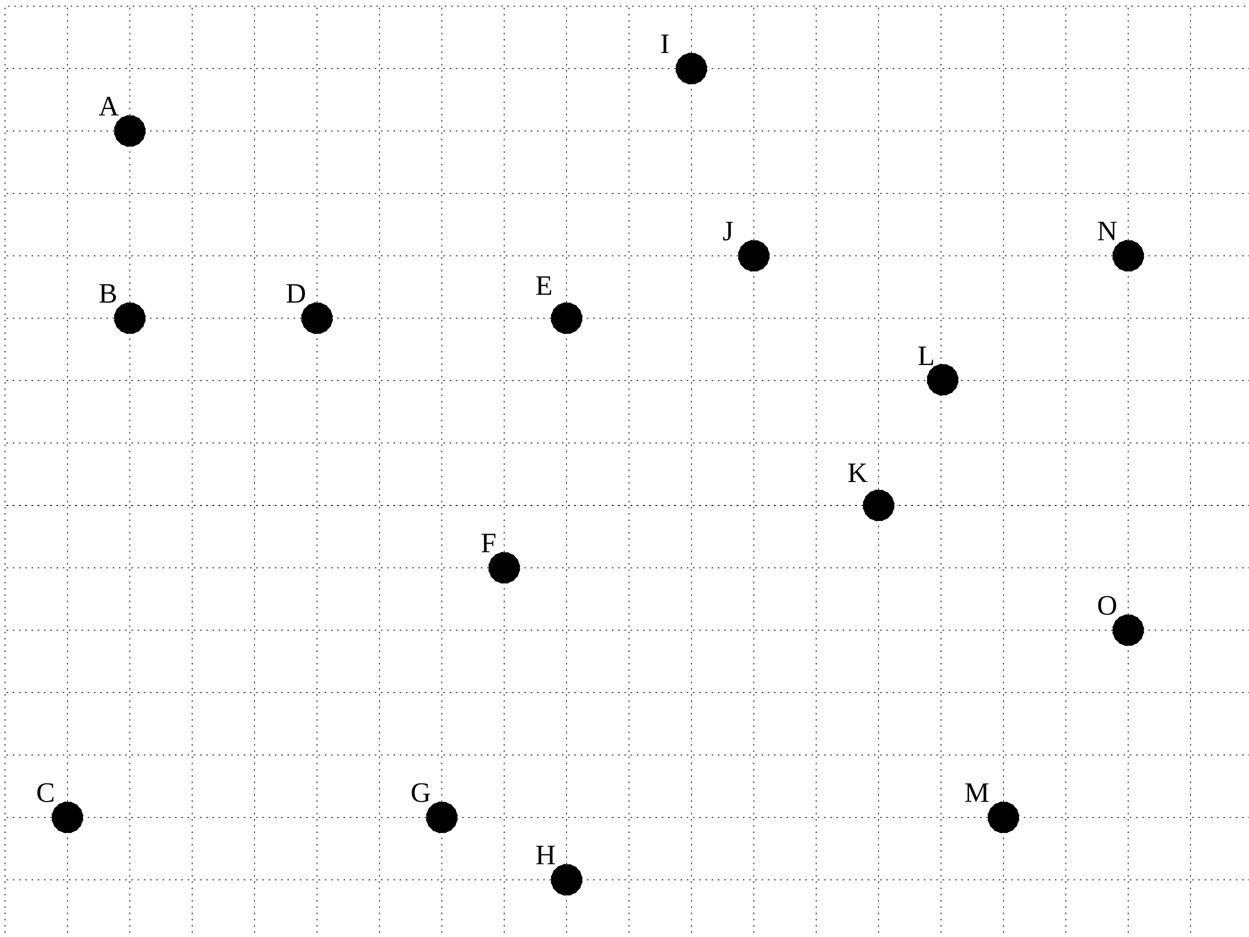
$\sqrt{37}\approx 6.1$

(b) Use Prim's algorithm (starting at **A**) to determine where they should put the wires.

- Draw the wires on the map below.
- For each wire give
  - its length
  - the order it was selected (in parentheses after the length)
- Give the total length of the wires.



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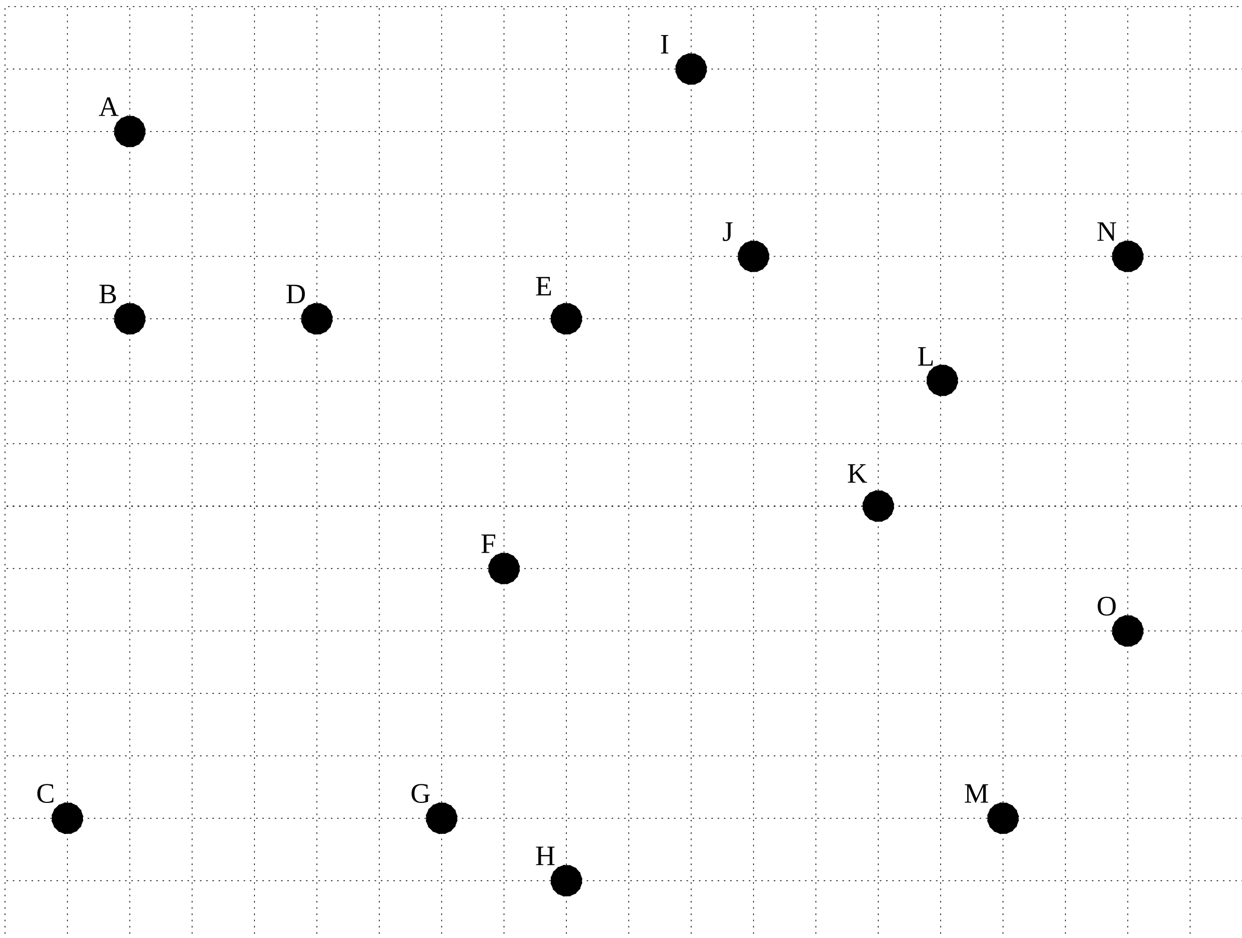
(c) Do your solutions have the same lengths? Should they? Explain.

7. The utility company for a rural town needs to add water lines to all of the houses. The water tower is at location **N**. The water lines can branch as necessary, but only at houses. There is a law that requires them to get the water to each house in as short of a length of pipe as possible so that everyone has fresh water. If it were not for technical and practical considerations, the law would require them to have a pipe running directly from the water tower to each house. However, they cannot have any pipes that are longer than 6.2 miles.

   (a) Subject to the given constraints, determine where they should put the pipes. (Hint: Use one of the algorithms from the book that you haven't used on this worksheet yet.)

   - Draw the pipes on the map below.
   - For each pipe give
     - its length
     - the order it was selected (in parentheses after the length)
   - For each house, give the total distance from **N**.



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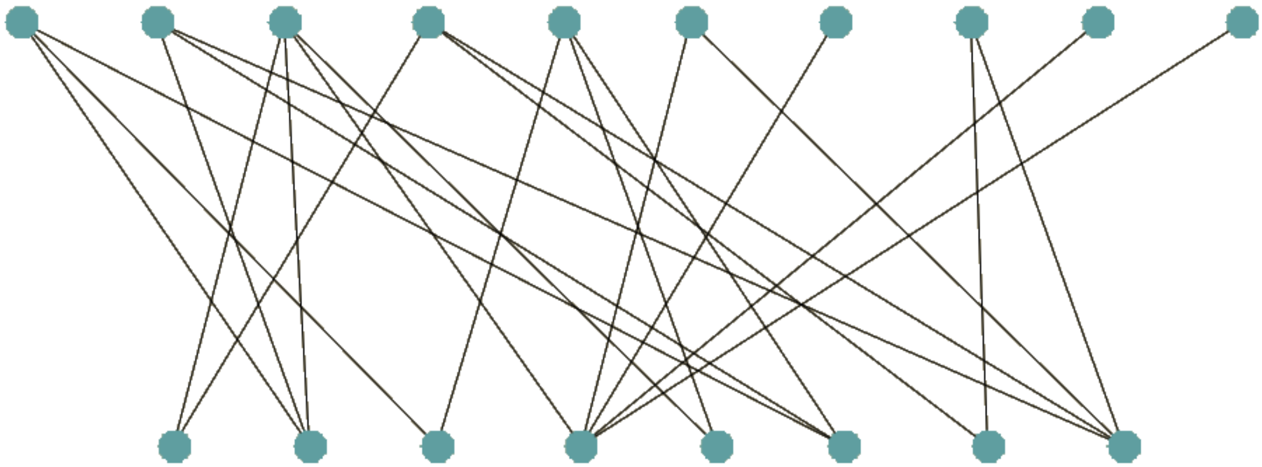
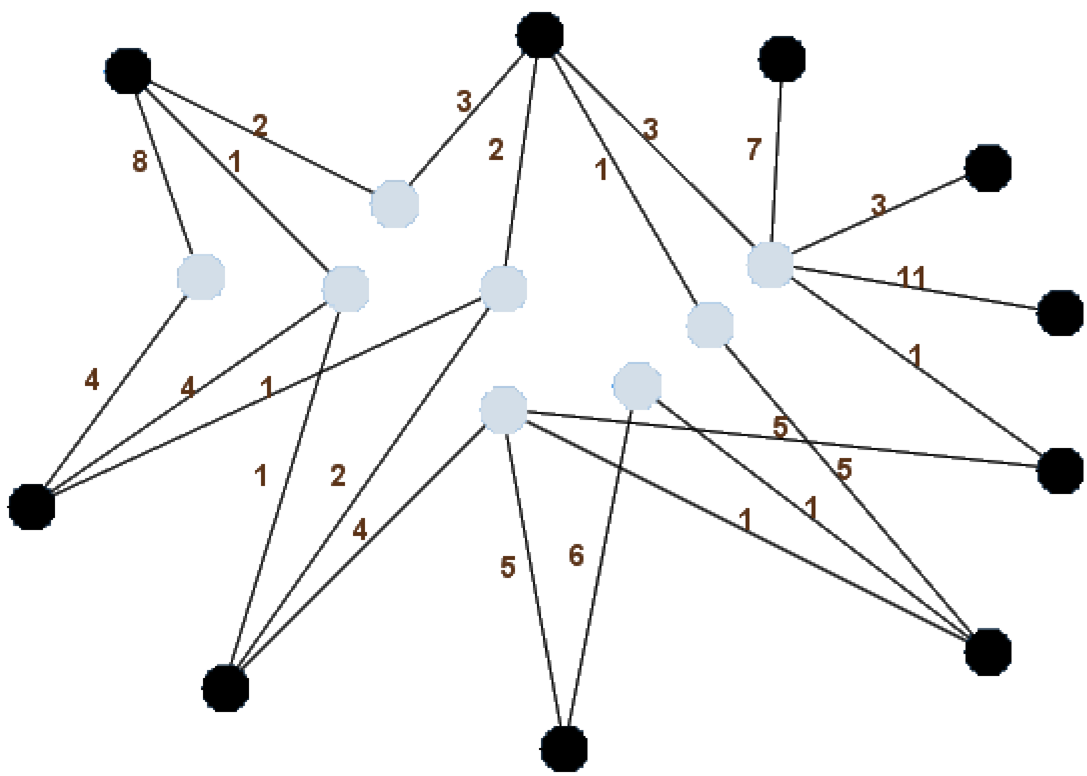
8. Ten friends are at a party eating pizza. There are only 8 slices left, and each person at the party is interested in one or more specific slices as indicated below (the people are at the top and the slices at the bottom). To be fair, nobody should get more than one slice, and they can't divide slices. Who should get which slices so that there are as few pieces left over as possible? Darken lines to indicate that a person should get a given slice.



9. The same ten people are at the same party with the same slices of pizza. But this time we want to take into account how much they are interested in each piece. A higher number means they desire that piece more than a piece with a lower number. We call this number the *utility*, and this time we are interested in maximizing the *utility*. Again assuming at most one slice per person and no splitting slices, darken lines to indicate who gets which slices.



10. **For discussion:** Answer each of the following for both of the previous problems.

    (a) What algorithm did you use?

    (b) Would it be easier to solve if a person could have more than one slice?

    (c) Would it be easier to solve if slices can be divided?