A. Instantaneous rate of change

Example

1. Let $P(t)$ represent the world population t minutes after midnight on January 1, 2012. Given that $P(0) = 7,028,734,178$ and that $P'(0) = 156$, estimate the population at the end of the month.
2. Let $M(v)$ represent the mileage (in mpg) of a car traveling at speed v (in mi/h). If $M(55) = 28$ and $M'(55) = -0.2$, estimate $M(65)$.

Remark

These examples utilize the approximation

$$f'(c) \approx \frac{f(c+h) - f(c)}{h} \iff f(c+h) \approx f(c) + f'(c) \cdot h$$

2.2 Interpretations of the derivative

A. Instantaneous rate of change

Remark

Let $s(t)$ represent the position s of an object moving in a straight line at time t.

1. The velocity of the object is $v(t) = s'(t)$.
2. The acceleration of the object is $a(t) = v'(t) = s''(t)$.

Example

Let $s(t) = t^2 - 3t$ describe the position (in m) of an object moving along a straight line as a function of time (in sec).

1. What is its position at $t = 2$? ____________________
2. What is its velocity at $t = 2$? ____________________
3. What is its acceleration at $t = 2$? ____________________
4. When is the object moving forward? ____________________

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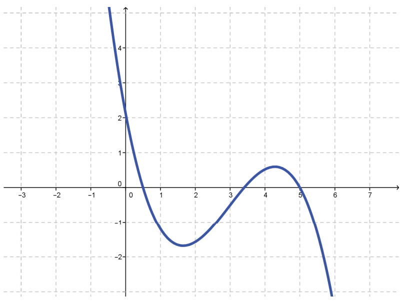
1. What is its position at $t = 2$? $s(2) = -2$ m
2. What is its velocity at $t = 2$? $v(2) = 1$ m/s
3. What is its acceleration at $t = 2$? $a(2) = 2$ m/s^2
4. When is the object moving forward? $(1.5, \infty)$

2.2 Interpretations of the derivative

B. Slope of the tangent line

Thinking of the derivative as the slope of a tangent line allows us to:

1. Compare (instantaneous) rates of change
   - e.g. How much faster is $x^2 + 1$ changing at $x = 2$ than at $x = 1$?
2. Sketch a graph of the derivative given a graph of the function
   - e.g. Suppose the graph of the function is



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   - e.g. Estimate $\sqrt{5}$

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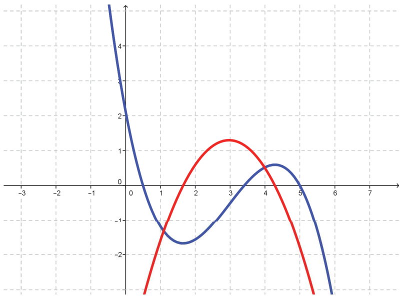

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2.2 Interpretations of the derivative

Just checking. . . .

1. What functions have a constant rate of change?
2. Given $f(5) = 9$ and $f'(5) = -0.3$, approximate $f(6)$.
3. The height H (in feet) of Lake Macatawa is is recorded t hours after midnight on May 1. What are the units of $H'(t)$? What does $H'(17) = -1/120$ mean?
4. Numerically approximate the value of $f'(4)$ for $f(x) = \ln x$.
5. Use the definition of the derivative to compute $f'(x)$ for $f(x) = (x - 2)^3$.