## 2.3 Differentation rules!

### Just checking. . . .

1. Differentiate whichever functions you can using the differentiation rules we've discussed so far.

   a. $f(x) = 3/x^2$
   b. $g(x) = 3/(x+1)^2$
   c. $h(x) = (3x^2+1)/x^3$
   d. $i(x) = 3x^3/(x^3+2)$
   e. $j(x) = \sqrt[3]{x}$
   f. $k(x) = \sqrt{x+1}$
   g. $\ell(x) = \sqrt[4]{x} + 3$
   h. $m(x) = 2e^x$
   i. $n(x) = e^{2x}$
   j. $p(x) = xe^2$
   k. $q(x) = \ln(x^2)$
   l. $r(x) = 2\sin x$
   m. $s(x) = \sin(2x)$
   n. $t(x) = \sin x \cos x$

2. Where does the line that is tangent to $f(x) = e^x + 3$ at $x = 0$ intersect the $x$-axis?
3. Approximate $e^{0.1}$ using an appropriate tangent line.

## 2.4 The product and quotient rules

### Theorem

(The Product Rule)
Let $f$ and $g$ be differentiable functions on an open interval $I$. Then $fg$ is a differentiable function on $I$, and

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

### Example

Find the derivatives of the following functions.

1. $f(x) = 3x\sin x$
   $f'(x) =$ ____________________
2. $f(x) = xe^x$
   $f'(x) =$ ____________________
3. $f(x) = x\ln x - x$
   $f'(x) =$ ____________________
4. $f(x) = (x+1)(3x^2-2)$
   $f'(x) =$ ____________________

## 2.4 The product and quotient rules

### Theorem

(The Product Rule)

Let $f$ and $g$ be differentiable functions on an open interval $I$. Then $fg$ is a differentiable function on $I$, and

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

### Example

Find the derivatives of the following functions.

1. $f(x) = 3x\sin x$

   $f'(x) = 3\sin x + 3x\cos x$
2. $f(x) = xe^x$

   $f'(x) = e^x + xe^x$
3. $f(x) = x\ln x - x$

   $f'(x) = \ln x$
4. $f(x) = (x+1)(3x^2-2)$

   $f'(x) = 9x^2 + 6x - 2$

## 2.4 The product and quotient rules

### Theorem

(The Quotient Rule)

Let $f$ and $g$ be differentiable functions on an open interval $I$, and suppose $g(x) \neq 0$ for all $x$ in $I$. Then $f/g$ is differentiable on $I$ and

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

### Example

Find the derivatives of the following functions.

1. $f(x) = (x^2+3)/x$
2. $f'(x) =$ ______________________
3. $f(x) = (x^2+3)/(x+1)$
4. $f'(x) =$ ______________________
5. $f(x) = x^2/(x+1)$
6. $f'(x) =$ ______________________
7. $f(x) = \tan x$
8. $f'(x) =$ ______________________

### Theorem

(The Quotient Rule)
Let $f$ and $g$ be differentiable functions on an open interval $I$, and suppose $g(x) \neq 0$ for all $x$ in $I$. Then $f/g$ is differentiable on $I$ and

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

### Example

Find the derivatives of the following functions.

1. $f(x) = (x^2 + 3)/x$
2. $f'(x) = 1 - 3/x^2$
3. $f(x) = (x^2 + 3)/(x + 1)$
4. $f'(x) = \frac{x^2+2x-3}{(x+1)^2}$
5. $f(x) = x^2/(x + 1)$
6. $f'(x) = \frac{x^2+2x}{(x+1)^2}$
7. $f(x) = \tan x$
8. $f'(x) = \sec^2 x$



### Theorem

Derivatives of trigonometric functions

1. $\frac{d}{dx}\left(\sin x\right) = \cos x$
2. $\frac{d}{dx}\left(\tan x\right) = \sec^2 x$
3. $\frac{d}{dx}\left(\sec x\right) = \sec x \tan x$
4. $\frac{d}{dx}\left(\cos x\right) = -\sin x$
5. $\frac{d}{dx}\left(\cot x\right) = -\csc^2 x$
6. $\frac{d}{dx}\left(\csc x\right) = -\csc x \cot x$

# 2.4 The product and quotient rules

### Just checking. . . .

1. True or false. The derivatives of the trigonometric "co-" functions (i.e. the ones that start with "co-") have minus signs in them.
2. Find the values of $x$ in $[-1, 1]$ where the tangent line to $f(x) = x \sin x$ is horizontal.
3. Find an equation of the normal line to $f(x) = \frac{x^2}{x-1}$ at $(2, 4)$.
4. Find the derivative of $xe^x \sin x$.
5. Find the derivative of $\sin x \csc x$.

# 2.5 The chain rule

### Example

Find the derivatives of the following functions.

1. $F_2(x) = (1-x)^2$. $F_2'(x) =$ ______________________________
2. $F_3(x) = (1-x)^3$. $F_3'(x) =$ ______________________________
3. $F_4(x) = (1-x)^4$. $F_4'(x) =$ ______________________________

### Remark

Notice that each of the functions is a composition $F_n(x) = f_n(g(x))$, where the "inner piece" is $g(x) = 1 - x$ and the "outer piece" is $f_n(x) = x^n$ (for $n = 2, 3, 4$). Notice, too, that we can differentiate each "piece." The chain rule tells us how to put the derivatives of these pieces together to get the derivative of a composition.