Example

1. If $y = x$, what is $\frac{d}{dx}\left(y^3\right)$?
2. If $y = \cos x$, what is $\frac{d}{dx}\left(y^3\right)$?
3. If $y = y(x)$, what is $\frac{d}{dx}\left(y^3\right)$?

Remark

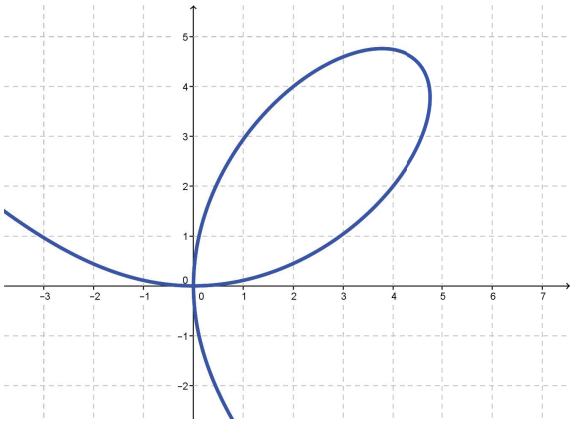
- When we know how y is related to x, we can differentiate any function of y with respect to x using the chain rule.
- When we do *not* know how y is related to x, we can still differentiate any function of y with respect to x using the chain rule – but the exact dependence of y on x (i.e. dy/dx) will remain implicit, since we can't determine explicitly what dy/dx is.

2.6 Implicit differentiation

Example

Any equation in x and y determines a curve in the xy-plane: the graph of an equation in x and y is just the set of points (x, y) that satisfy the equation. For example, the graph of the equation $x^3 + y^3 = 9xy$ is shown below at right.

1. Verify that $(2, 4)$ lies on the curve $x^3 + y^3 = 9xy$.
2. Use the graph to estimate $\frac{dy}{dx}\Big|_{(2,4)}$.
3. Treating y as an unknown function of x locally, which we write as $[y = y(x)]$, compute $\frac{dy}{dx}\Big|_{(2,4)}$.



$x^3 + y^3 = 9xy$

2.6 Implicit differentiation

Example

Find dy/dx for the following curves.

1. $x^{2/5} + y^{2/5} = 1$

   $dy/dx =$ _____

2. $\dfrac{x^2 + y}{x + y^2} = 17$

   $dy/dx =$ _____

3. $\ln(x^2 + xy + y^2) = 1$

   $dy/dx =$ _____

4. $\dfrac{\cos(x^2y^2) + y^3}{x + y} = 1$

   $dy/dx =$ _____

Example

Find an equation for the line tangent to the circle $(x - 1)^2 + (y + 2)^2 = 1$ at the point $(3/2, \sqrt{3}/2 - 2)$ in two ways:

- by using implicit differentiation
- by first solving for y as a function of x and then differentiating

2.6 Implicit differentiation

Example

Find dy/dx for the following curves.

1. $x^{2/5} + y^{2/5} = 1$

   $dy/dx = -\dfrac{y^{3/5}}{x^{3/5}}$

2. $\dfrac{x^2 + y}{x + y^2} = 17$

   $dy/dx = \dfrac{2x - 17}{34y - 1}$

3. $\ln(x^2 + xy + y^2) = 1$

   $dy/dx = -(2x + y)/(x + 2y)$

4. $\dfrac{\cos(x^2y^2) + y^3}{x + y} = 1$

   $dy/dx = -\dfrac{2xy^2 \sin(x^2y^2) + 1}{2x^2y \sin(x^2y^2) - 3y + 1}$

Example

Find an equation for the line tangent to the circle $(x - 1)^2 + (y + 2)^2 = 1$ at the point $(3/2, \sqrt{3}/2 - 2)$ in two ways:

- by using implicit differentiation
- by first solving for y as a function of x and then differentiating

2.6 Implicit differentiation

Theorem

Let r be a real number. Then

$$\frac{d}{dx}(x^r) = rx^{r-1}$$

Implicit differentiation and second derivatives

Example

Find d^2y/dx^2 where $x^2 + y^2 = 4$.

Logarithmic differentiation

Example

Find dy/dx for the following functions.

1. $y = \dfrac{(x-2)^3\sqrt{3x+1}}{(2x+5)^4}$
2. $y = x^{\sin x}$

2.6 Implicit differentiation

Just checking. . . .

1. Find dy/dx for the following implicitly defined functions.
   a. $xy = 1$
   b. $x^2y^2 = 1$
   c. $\sin(xy) = 1$
   d. $\ln(xy) = 1$
2. Find dy/dx for $x^2 \tan y = 50$.
3. Find an equation for the line tangent to $y = (2x)^{x^2}$ at $x = 1$.
4. Find d^2y/dx^2 for $\cos x + \sin y = 1$.
5. Using the defintion of the derivative, compute $f'(x)$ for $f(x) = \sqrt{3x+1}$, and write an equation for the line tangent to $y = f(x)$ at $(1, 2)$.