

3.1 Extreme values

Just checking. . . .

- ① True or false. If c is a critical value of a function f , then f has either a relative maximum or a relative minimum at $x = c$.
- ② Find the extreme values of $f(x) = x^3 - \frac{9}{2}x^2 - 30x + 3$ on $[0, 6]$.
- ③ Let $f(x) = x^3 + x$. Evaluate $\lim_{s \rightarrow 0} \frac{f(x+s) - f(x)}{s}$.
- ④ Find $\frac{d}{dx} (\sin^{-1}(e^{2x}))$.
- ⑤ Find $\frac{d}{dx} (3^{x^2})$.

3.2 The mean value theorem

Remark

- The average rate of change of f over the interval $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a}$$

- The instantaneous rate of change in f at c is $f'(c)$.

Theorem

The Mean Value Theorem

Let $f(x)$ be continuous on $[a, b]$ and differentiable on (a, b) . Then there exists a value of c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

3.2 The mean value theorem

Proof of the mean value theorem

Theorem

Rolle's theorem

Let f be continuous on $[a, b]$ and differentiable on (a, b) , and suppose $f(a) = f(b) = 0$. Then there is some c in (a, b) such that $f'(c) = 0$.

Proof.

3.2 The mean value theorem

Proof of the mean value theorem

Theorem

The Mean Value Theorem

Let $f(x)$ be continuous on $[a, b]$ and differentiable on (a, b) . Then there exists a value of c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Proof.

3.2 The mean value theorem

Example

Consider $f(x) = \ln x$ on $[1, 5]$. Find the value of c in $(1, 5)$ where $f'(c)$ equals the average rate of change of $\ln x$ over $[1, 5]$.

Example

Consider $f(x) = (x - 1)^{2/3}$ on $[0, 9]$. Show that there is no value of c in $(0, 9)$ where the instantaneous rate of change in f at c equals the average rate of change in f over $[0, 9]$. Does this contradict the mean value theorem? Why or why not?

3.2 The mean value theorem

Just checking. . . .

- 1 Find the value of c in $(-5, 2)$ where the average rate of change in $f(x) = 2x^3 - 5x^2 + 6x + 1$ equals the instantaneous rate of change in f at c .
- 2 Find the extreme values of $f(x) = x^2 - 3x + 9$ on $[-2, 5]$.
- 3 Find $\frac{d}{dx} ((\cos x)^{2x})$.
- 4 Does $f(x) = \frac{x+2}{2x-3}$ have an inverse? If so, find it. If not, state why not.
- 5 Find $\frac{d}{dx} (\sin(e^{\sqrt{3x+5}}))$.