Just checking. . . .

1. Find the root of $f(x) = x + \ln x$, accurate to two decimal places.
2. Where is the function $f(x) = 2x^3 + x^2 - x + 3$ increasing, and where is it decreasing?
3. Find dy/dx if $x^{2/3} + y^{2/3} = 1$.
4. Find dy/dx if $y = \csc^{-1}(3x)$.
5. How close to 1 does x need to be in order that $\ln x$ be within 0.1 of $\ln 1 = 0$?

4.2 Related Rates

Method

1. Draw a picture and label it with variables and constants.

   Anything that varies in the problem should be a variable; only if a quantity remains constant throughout the problem can it be labeled as a constant.

2. Write down the rate(s) you know and the rate you want to find out *as derivatives*.

3. Find an equation that relates the variables involved in the known rate(s) and desired rate.

4. Apply d/dt to the above equation to differentiate with respect to time (using implicit differentiation), and plug in instantaneous values of variables and valus of known rates to solve for the desired rate.

Example

1 Water is being pumped into an empty spherical tank at 10 cubic feet per minute starting at time $t = 0$ minutes. The radius of the tank is 8 feet. At time $t = 2$ minutes, how fast is the depth of the water increasing? Note that the volume of a spherical cap of radius R and depth y is $V = (\pi/3)y^2(3R - y)$.

2 A radar station is tracking a plane. The plane is flying straight and level at 5000 feet and on a course that will take it directly over the radar dish. Its speed is 880 feet per second. How fast is the angle that the line connecting the dish to the plane makes with the ground changing when the plane is 10,000 feet away from the dish?

Example

1 The volume of a spherical balloon is increasing at 1/2 cubic feet per minute. How fast is the radius increasing when the volume is 1/4 cubic feet? How about the surface area?

Example

① A pebble thrown into the pond produces circular ripples whose radii are growing at a rate of 5 in/s. At what rate is the circumference a ripple growing when the ripple has a radius of 10 in? At what rate is the area growing at that time?

② A police officer is driving north at 30 mph on US 31 when she sees you cross the highway heading east on Port Sheldon Road. Suspecting that you might be speeding, she turns on her radar gun to get a reading. By using landmarks, she estimates that you and she are both 1/2 mile from the intersection when she takes a radar gun reading of 25 mph. Does she turn onto Port Sheldon and flip on her lights to pull you over, or were you observing the posted speed limit of 55 mph on Port Sheldon Road?

Just checking. . . .

① Find $\lim\limits_{x \to 2} \dfrac{\frac{3}{x} - \frac{3}{2}}{x - 2}$.

② Find the value of c that makes the function
$$f(x) = \left\{ \begin{array}{ll} 3x - 2 & x \leq 1 \\ 2x^2 + c & x < 1 \end{array} \right\}$$
continuous at $x = 1$. With this value of c, is f differentiable at $x = 1$?

③ Is there a value of m that makes the function
$$f(x) = \left\{ \begin{array}{ll} \sin(3x - \pi/6) & x < 0 \\ mx - 1/2 & \end{array} \right\}$$
continuous at $x = 0$? differentiable at $x = 0$?

④ Find dy/dx if $y = 3^{4x}$.

⑤ At what values of x, if any, does the function $f(x) = \frac{2x^2+6}{x-1}$ have a horizontal tangent? Give equations for any horizontal tangents, horizontal asymptotes, and vertical asymptotes.