5.1 Antiderivatives and indefinite integration

Just checking. . . .

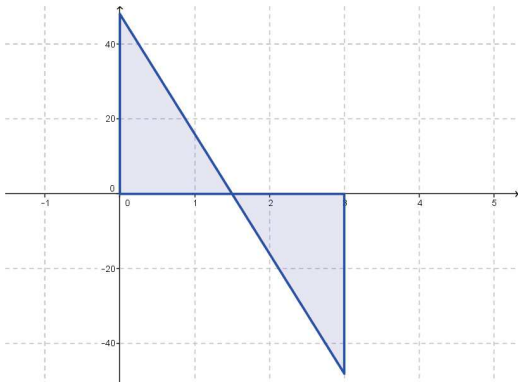
1. Find $\int (x^2 + 3)(x - 2)\, dx$.
2. Given $y = x^2 e^x \cos x$, find dy.
3. Find dy/dx if $x^3 + xy - \cos y = 5$.
4. Find the area of the largest rectangle that can be placed between the x-axis and the parabola $y = 9 - x^2$.
5. Find $f(x)$ if $f''(x) = x$ and $f(0) = 1$ and $f(2) = 3$.

5.2 The definite integral

Example

The velocity of a baseball moving straight up and down under the acceleration of gravity is $v(t) = -32t + 48$, where time t is given in seconds and velocity v is in ft/s. When $t = 0$, the baseball had a height of 0 ft.

1. What was the initial velocity of the baseball?
2. What was the maximum height of the baseball?
3. What was the height of the baseball at time $t = 2$?



Definition

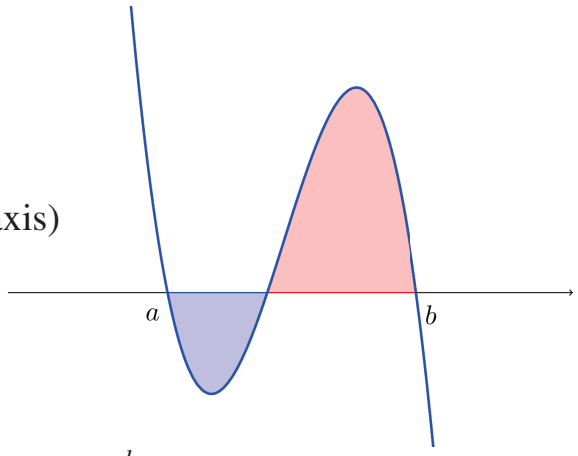
Let $y = f(x)$ be defined on a closed interval $[a, b]$. The **total signed area** between $y = f(x)$ and the x-axis from $x = a$ to $x = b$ is

(area above x-axis)−(area below x-axis)

The **definite integral**

$$\int_a^b f(x)\,dx$$

computes the total signed area.

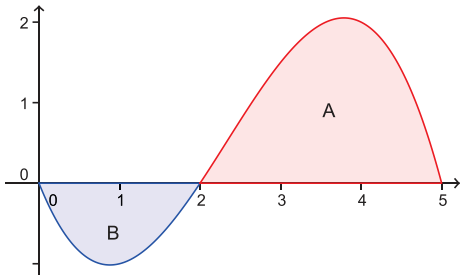


$$\int_a^b f(x)\,dx = \text{red} - \text{blue}$$

5.2 The definite integral

Example

Suppose the area of region A is 4 and the area of region B is 1.



1. $\int_0^2 f(x)\,dx =$ _____
2. $\int_3^3 f(x)\,dx =$ _____
3. $\int_2^5 f(x)\,dx =$ _____
4. $\int_0^5 f(x)\,dx =$ _____
5. $\int_0^2 |f(x)|\,dx =$ _____
6. $\int_2^5 -f(x)\,dx =$ _____

Example

Evaluate each integral by interpreting it in terms of signed area.

1. $\int_1^3 7\,dx =$ _____
2. $\int_1^3 (1+2x)\,dx =$ _____
3. $\int_{-2}^3 |x-2|\,dx =$ _____
4. $\int_{-2}^3 |x|-2\,dx =$ _____
5. $\int_{-5}^5 \sqrt{25-x^2}\,dx =$ _____
6. $\int_0^5 -\sqrt{25-x^2}\,dx =$ _____

5.2 The definite integral

Theorem

Let f and g be defined on a closed interval I that contains the values a, b and c, and let k be a constant.

1. $\int_a^a f(x)\,dx = 0$
2. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$
3. $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
4. $\int_a^b (f(x) \pm g(x))\;dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
5. $\int_a^b k \cdot f(x)\,dx = f \cdot \int_a^b f(x)\,dx$

5.2 The definite integral

Just checking. . . .

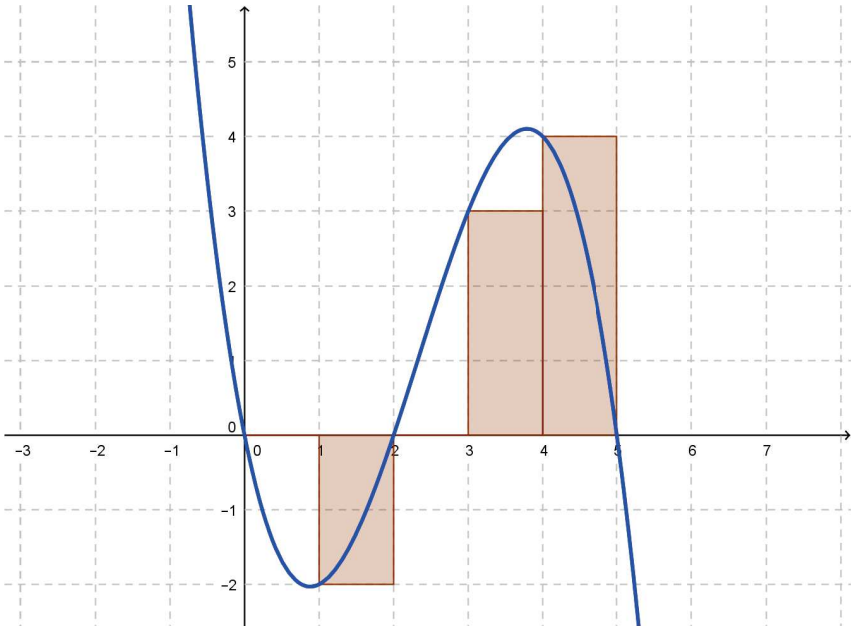
Suppose

- $\int_0^2 f(x)\,dx = 5$
- $\int_0^3 f(x)\,dx = 7$
- $\int_0^2 g(x)\,dx = -3$
- $\int_2^3 g(x)\,dx = 5$

1. Find $\int_0^2 f(x) + g(x)\,dx$
2. Find $\int_2^3 3f(x) - 2g(x)\,dx$
3. Find $\int_0^3 4g(x) - x + 1\,dx$
4. Find values for a and b such that $\int_0^3 af(x) + bg(x)\,dx = 0$
5. Find $\int_{-\pi}^{\pi} \sin x\,dx$

5.3 Riemann sums

Idea: add up areas of rectangles to approximate



5 rectangles, left endpoints