5.2 The definite integral

Just checking. . . .

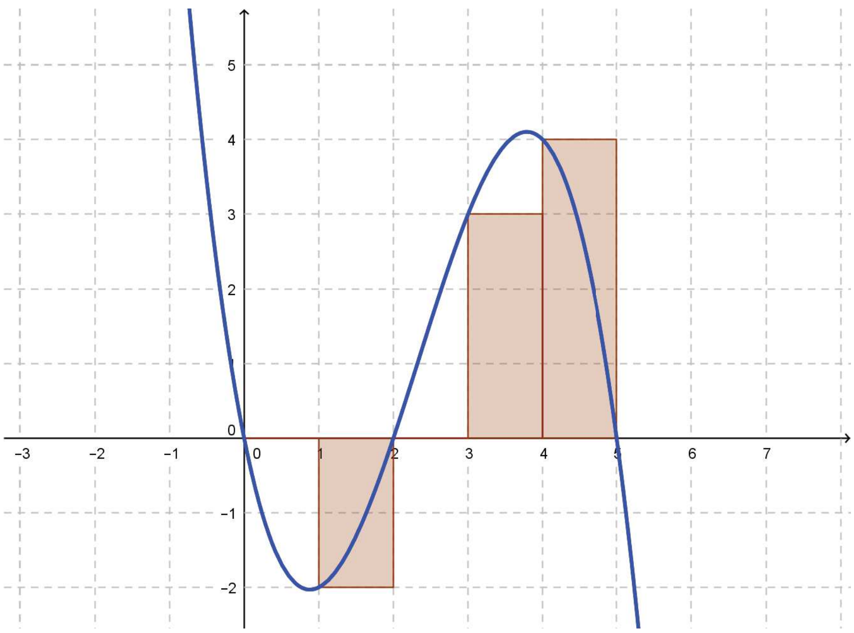
Suppose

- $\displaystyle\int_0^2 f(x)\,dx = 5$
- $\displaystyle\int_0^3 f(x)\,dx = 7$
- $\displaystyle\int_0^2 g(x)\,dx = -3$
- $\displaystyle\int_2^3 g(x)\,dx = 5$

1. Find $\displaystyle\int_0^2 f(x) + g(x)\,dx$
2. Find $\displaystyle\int_2^3 3f(x) - 2g(x)\,dx$
3. Find $\displaystyle\int_0^3 4g(x) - x + 1\,dx$
4. Find values for a and b such that $\displaystyle\int_0^3 af(x) + bg(x)\,dx = 0$
5. Find $\displaystyle\int_{-\pi}^{\pi} \sin x\,dx$

5.3 Riemann sums

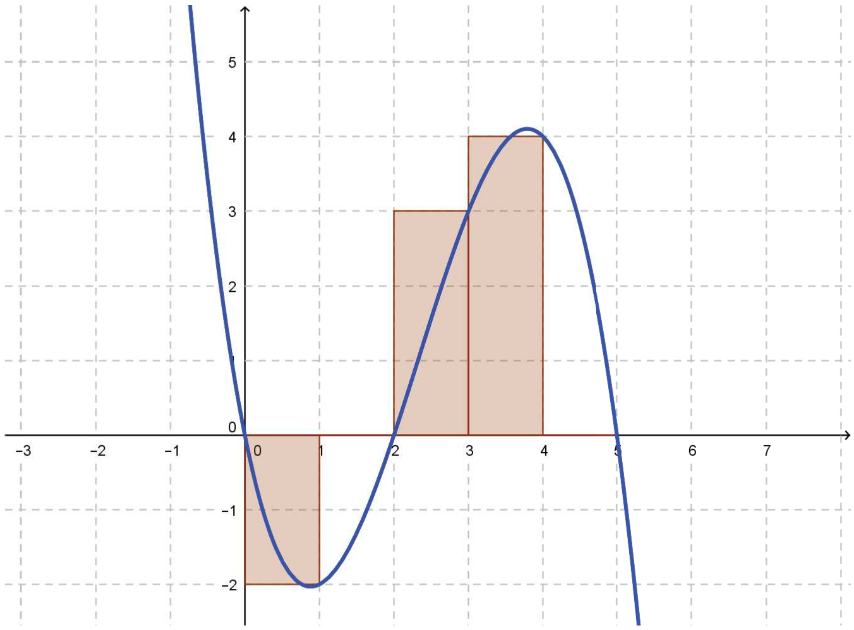
Idea: add up areas of rectangles to approximate



5 rectangles, left endpoints

5.3 Riemann sums

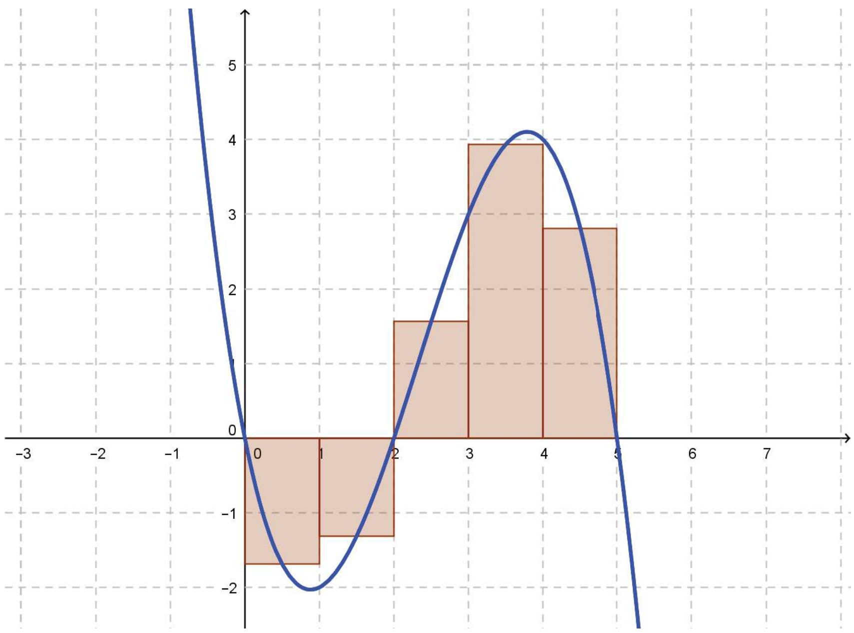
Idea: add up areas of rectangles to approximate



5 rectangles, right endpoints

5.3 Riemann sums

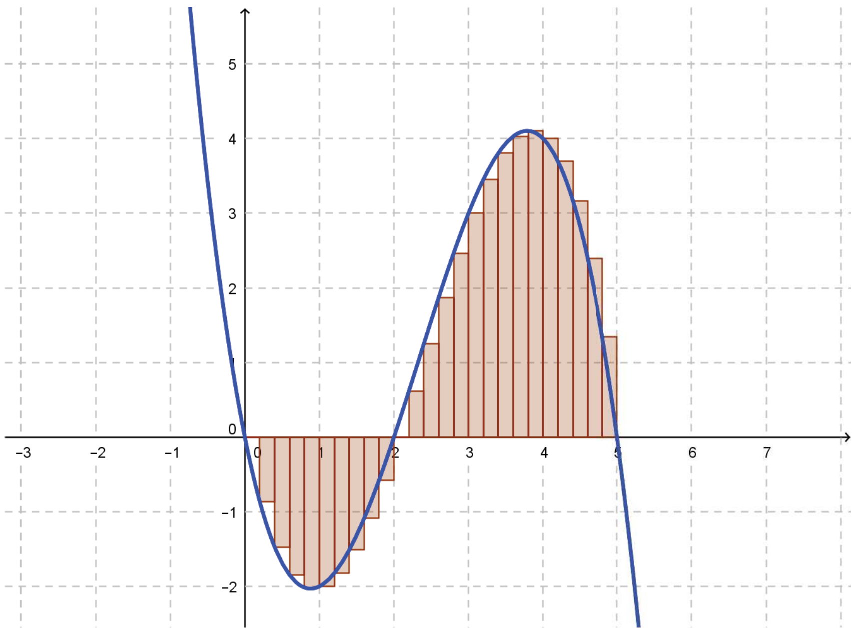
Idea: add up areas of rectangles to approximate



5 rectangles, midpoints

5.3 Riemann sums

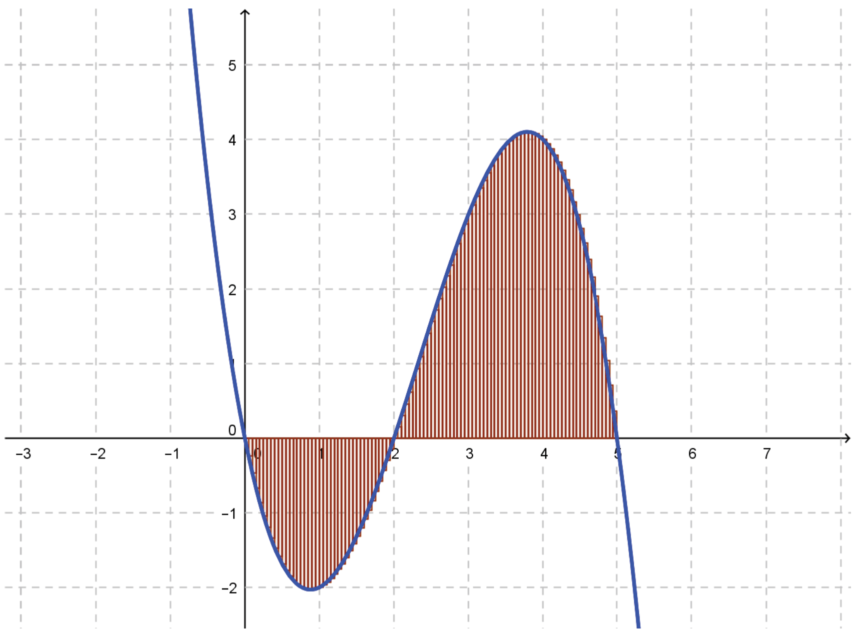
Idea: add up areas of rectangles to approximate



25 rectangles, left endpoints

5.3 Riemann sums

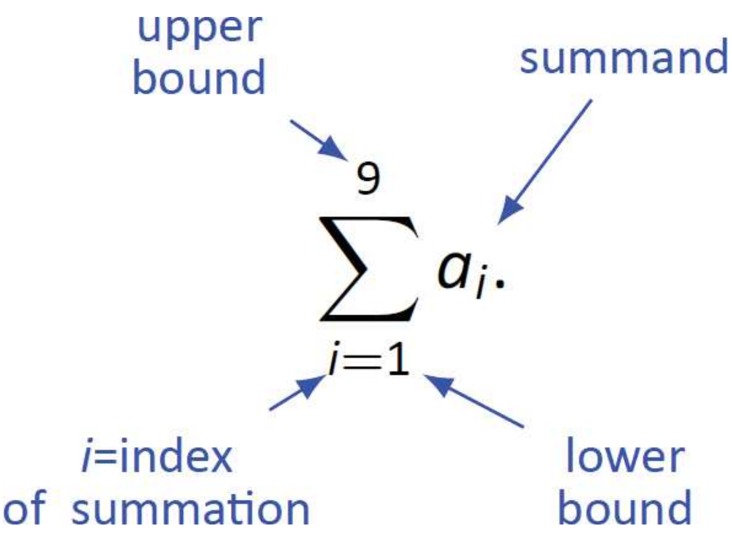
Idea: add up areas of rectangles to approximate



100 rectangles, left endpoints

5.3 Riemann sums

Sigma notation



$$\sum_{i=1}^{9} a_i = a_1 + a_2 + a_3 + \cdots + a_9$$

5.3 Riemann sums

Sigma notation

Example

Expand the following summations and evaluate the sums.

1. $\sum_{k=2}^{5} 3k =$ ________________
2. $\sum_{j=-3}^{1} j^2 =$ ________________
3. $\sum_{i=0}^{4} i/3 =$ ________________
4. $\sum_{n=1}^{4} n^2 - n =$ ________________

Sigma notation

Theorem

Formal properties

1. $$\sum_{i=m}^{n}(a_i \pm b_i) = \sum_{i=m}^{n} a_i \pm \sum_{i=m}^{n} b_i$$
2. $$\sum_{i=m}^{n} c \cdot a_i = c \cdot \sum_{i=m}^{n} a_i$$
3. $$\sum_{i=m}^{k} a_i + \sum_{i=k+1}^{n} a_i = \sum_{i=m}^{n} a_i$$

Common sums

1. $$\sum_{i=1}^{n} 1 = n$$
2. $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
3. $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$
4. $$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Example

Compute $\int_1^3 x^2\, dx$.

1. Divide $[a, b]$ into n subintervals of width $\Delta x = \frac{b-a}{n}$.

   $\Delta x =$ ____________________

2. Find the right endpoint of the i^{th} subinterval $x_i = a + i\Delta x$.

   $x_i =$ ____________________

3. Use the right endpoint to compute the height of the i^{th} rectangle $f(x_i)$.

   $f(x_i) =$ ____________________

4. Add the areas of all n rectangles to compute the n^{th} Riemann sum R_n:

   $$R_n = \sum_{i=1}^{n} f(x_i) \cdot \Delta x = \_\_\_\_\_\_\_\_\_\_$$

5. Take the limit as $n \to \infty$ (so that $\Delta x \to 0$):

   $$\int_1^3 f(x)\, dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \_\_\_\_\_\_\_\_\_\_$$

Example

1. Compute $\int_1^3 x^2 + 1\, dx$. (Hint: cheat!)

   $\int_1^3 x^2 + 1\, dx =$ ______________________________

2. Compute $\int_0^4 -x^2 + 4x\, dx$. (Hint: don't cheat!)

   $\int_0^4 4x - x^2\, dx =$ ______________________________

Remark

1. Compute $\int x^2 + 1\, dx = F(x)$ and evaluate $F(3)$ and $F(1)$.
2. Compute $\int 4x - x^2\, dx = F(x)$ and evaluate $F(4)$ and $F(0)$.

Just checking. . . .

1. Write $1 + 4 + 9 + \cdots + 400$ in sigma notation.
2. Evaluate $\sum_{i=15}^{71} i$.
3. Find an antiderivative of $1/\sqrt{x}$.
4. Approximate $\int_1^3 \ln x\, dx$ using four rectangles of equal width and right endpoints.
5. Compute $\int_{-10}^{10} 5 - x\, dx$ using Riemann sums and check your answer using geometry.