Definition of the Definite Integral

Remark

Given a closed interval $[a, b]$. A *partition* of $[a, b]$ is a set of numbers $\mathcal{P} = \{x - 0, \ldots, x_n\}$ with $a = x_0 < x_1 < \cdots < x_n = b$ that divides $[a, b]$ into n subintervals. Suppose that all subintervals are of equal length, $\Delta x = (b - a)/n$.
A function f is *integrable* on $[a, b]$ if

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

exists independent of the choices of x_k^* in $[x_{k-1}, x_k]$. In this case, the *definite integral of f from a to b*, written

$$\int_a^b f(x)\,dx$$

is the value of that limit.

5.4 The Fundamental Theorem of Calculus

Theorem

Part 1 of the FTC

Let f be continuous on $[a, b]$ and let $F(x) = \displaystyle\int_a^x f(t)\,dt$. Then F is a differentiable function on (a, b) and

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Remark

1. $F(x) = \displaystyle\int_a^x f(t)\,dt$ is an antiderivative of f.
2. The rate at which area is being added under the graph of $y = f(x)$ at any point x is equal to the value of the function $f(x)$.