This is a student's writing assignment for 3 problems from the first assignment from Calc 1 in Fall 2011. Each question was graded on a 20 point scale (see the grading criteria elsewhere).

This assignment received a grade of 53/60. The first and third problem are very good. The second problem isn't perfect, but it is pretty good. See the last page for a detailed assignment of points.

Writing Problems (2.2)

#10 Problem: If f(1)=5, must $\lim_{x \to 1} f(x)$ exist? If it does, then must $\lim_{x \to 1} f(x)=5$? Can we conclude anything about $\lim_{x \to 1} f(x)$? Explain.

We are asked to determine if $\lim_{x \to 1} f(x)$ exists when f(1)=5 and whether or not we can actually conclude anything about $\lim_{x \to 1} f(x)$.

First off, we define the term "limit", resulting in: $\lim_{x \to x_0} f(x)= L$

Or in other terms, if f(x) is arbitrarily close to L for all x sufficiently close to x_0, we say that f approaches the L as x approaches x_0 (on either side of x_0).

When looking back at the problem, we notice that f(1) does indeed equal 5. So immediately, we might assume that $\lim_{x \to 1} f(x)$ does indeed exist and that it is equal to 5.

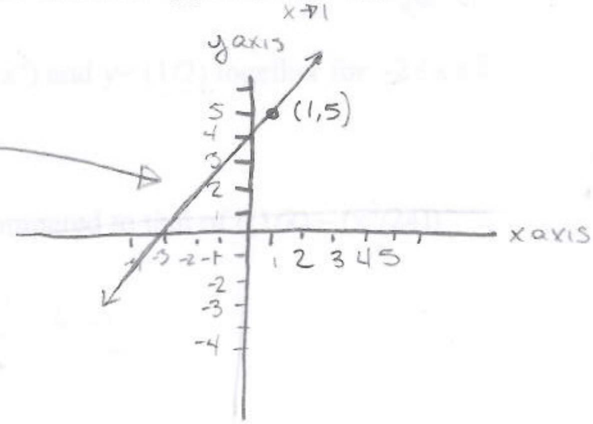
However, because we do not know what the actual function is, and if we approached $\lim_{x \to 1}$ f(x) from both sides of 1

$$\lim_{x \to 1^+} f(x) \quad \& \quad \lim_{x \to 1^-} f(x)$$

we cannot assume that $\lim_{x \to 1} f(x)$ does in fact exist. By looking at the following graphs and functions, we can see how $\lim_{x \to 1} f(x)$ might and might not exist, leaving uncertainty.

In this function and graph we see that f(1) does equal 5 and does approach $\lim_{x \to 1} f(x)=5$ from both sides of 1.

f(x) = x+4 $\quad f(1) = 1+4 = 5$



We plug in 1 for x which results in f(1)=5. When we take the limit of .9999 and 1.0001 (getting closer and closer to 1 from both sides) for f(x), we see that it gets closer and closer to 5, telling us that the limit of f(x) does equal 5.

$\lim_{x \to 1^+} (.9999+4) = 4.9999$

$\lim_{x \to 1^-} (1.0001+4) = 5.0001$

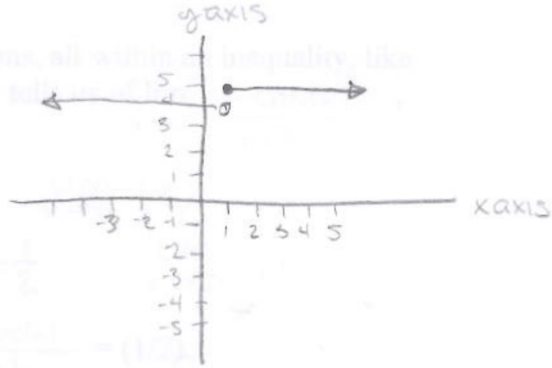
However, in this function and graph, we see how f(1)=5 but the $\lim_{x\to 1}$ f(x) does not exist due to a unit step function (f(x)). When we approach 1 from both sides, the limits $\lim_{x\to 1^+}$ f(x) & $\lim_{x\to 1^-}$ f(x) do not equal.

$$f(x)=\begin{cases}4 & x<1\\ 5 & x\geq 1\end{cases}$$

$$\lim_{x\to} (.9999) = 4$$

$$\lim_{x\to 1} (1.0001) = 5$$

When we plug in 1 for this function, we see that at f(1) it does equal 5, however when we find the limit from both sides, as x approaches .9999 and 1.0001, we see that the limits do not equal.



By looking at these two examples, we see that we cannot conclude anything about $\lim_{x\to 1}$ f(x) and whether or not it exists because, again, we see without the actual equation, we cannot assume $\lim_{x\to 1}$ f(x) exists even though f(1)=5.

#66 Problem: A) Suppose that the inequality

$$\frac{1}{2}-\frac{x^2}{24} < \frac{1-\cos(x)}{x^2} < \frac{1}{2}$$

Holds for values of x close to zero. What, if anything, does this tell you about $\lim_{x\to 1} \frac{1-\cos(x)}{x^2}$
Give reasons.

B) Graph the equations y= (1/2)- (x^2/24) , y= (1 – cos(x))/ x^2) and y= (1/2) together for $-2 \leq x \leq 2$.
Comment on the behavior of the graphs as x→ 0.

We are asked to determine what $\lim_{x\to 1} \frac{1-\cos(x)}{x^2}$ is compared to that of ((1/2) – (x^2/24)) and (1/2) in the inequality:

$$\frac{1}{2}-\frac{x^2}{24} < \frac{1-\cos(x)}{x^2} < \frac{1}{2}$$

We are then later asked to graph each of those three functions together on our calculator between $-2 \leq x \leq 2$ and comment on how the graphs behave as x→ 0.

To start off, we can look back to the Sandwich Theorem which is defined as:

"Suppose that $g(x) \le f(x) \le h(x)$ for all x in some interval containing c, except possibly at x=c itself. Suppose also that :

$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$ Then $\lim_{x \to c} f(x) = L$.

By looking at our problem, we see that we have 3 functions, all within an inequality, like the sandwich theorem. To determine what the inequality tells us of $\lim_{x \to 0} \frac{1-\cos(x)}{x^2}$,

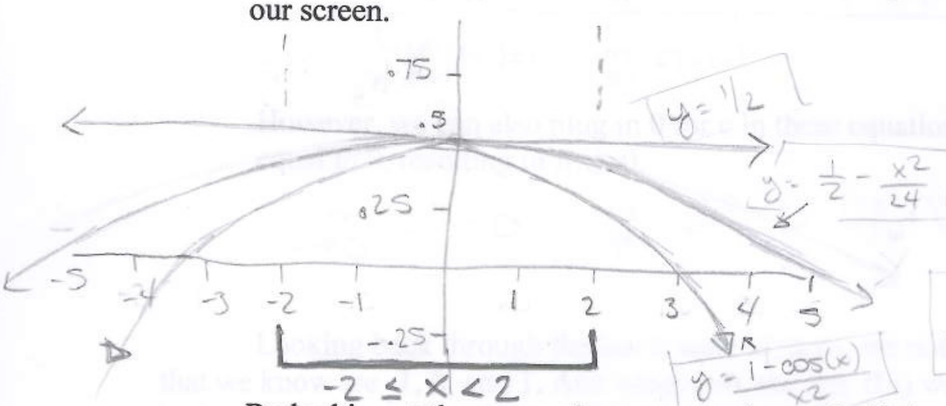
we apply the sandwich theorem. By taking the limits of both sides $\lim_{x \to 0} \left(\frac{1}{2} - \frac{x^2}{24}\right)$ $\lim_{x \to 0} \left(\frac{1}{2}\right)$

we find that they both equal (1/2). $\lim_{x \to 0} \left(\frac{1}{2} - \frac{0^2}{24}\right) = \frac{1}{2}$ $\lim_{x \to 0} \left(\frac{1}{2}\right)$

Therefore, the Sandwich Theorem implies that $\lim_{x \to 0} \frac{1-\cos(x)}{x^2}$ = (1/2).

This tells us the Sandwich Theorem does in fact show that as $x \to 0$ in all the 3 equations, the limit then equals (1/2).

B) When we plug in all 3 equations from the inequality between $-2 \le x \le 2$, we see this on our calculator graph screen.



$\lim_{x \to 0} \frac{1-\cos(x)}{x^2}$

$\lim_{x \to 0} \frac{1-\cos(.9999)}{(.9999)^2} = \lim_{x \to 0} \frac{1-\cos(1.0001)}{(1.0001)^2}$

By looking at these equations, we see that at 0, all three seem to be getting closer to (1/2) from both sides. In the case of $((1/2)-(x^2/24))$ and (1/2), we get (1/2) when we put 0 in for x. However, with $(1-\cos(x))/(x^2)$, we do not get a definite answer of (1/2) when we plug in 0 on our calculator. Therefore, when we take the limit of both sides as $x \to 0$, we see that the function does indeed get arbitrarily close to (1/2) as x approaches 0.

It can therefore be concluded that the limit of $(1-\cos(x))/(x^2)$ does equal (1/2), which is the same as the limits of $((1/2)-(x^2/24))$ and (1/2) as $x \to 0$. And when the equations are graphed, we see that all 3 seem to be moving towards (1/2) as $x \to 0$; even though the limit of $((1/2)-(x^2/24))$ does not equal (1/2) as $x \to 0$, it still gets arbitrarily close.

#77 Problem: If $x^4 \le f(x) \le x^2$ for x in [-1,1] and $x^2 \le f(x) \le x^4$ for $x < -1$, and $x>1$ at what points **c** do you automatically know $\lim_{x \to c} f(x)$? What can you say about the value of the limit at these points?

We are asked to determine at what points **c** do we automatically know the limit of f(x) as x→ c. We are then to determine what the limit value is of those points.

For this problem we can, again, look at the Sandwich Theorem:
"Suppose that $g(x) \le f(x) \le h(x)$ for all x in some interval containing c, except possibly at x=c itself. Suppose also that :

$\lim_{x \to c} g(x) = \lim_{x \to c} (h(x)) = L$ Then $\lim f(x) = L$.

When we take the limit of both sides of the inequality using the interval numbers [-1,1], we end up with:

$\lim_{x \to -1} (-1)^4 = 1$ $\lim_{x \to -1} (-1)^2 = 1$

$\lim_{x \to 1} (1)^4 = 1$ $\lim_{x \to 1} (1)^2 = 1$

By using the sandwich theorem, it can be assumed then that the value of f(x) would be equal to 1, for both interval numbers [-1,1] plugged in.

$\lim_{x \to -1} f(x) = 1$ $\lim_{x \to 1} f(x) = 1$

However, we can also plug in 0 for c in these equations, as both sides would then be equal to 0, resulting in f(x)=0.

$\lim_{x \to 0} (0)^4 = 0$ $\lim_{x \to 0} (0)^2 = 0$ $\lim_{x \to 0} f(x) = 0$

Again, applying the Sandwich Theorem, we find that as $x \to 0$ the limit of f(x) would equal 0.

Looking back through the last couple of steps, we notice then that the automatic points c that we know are -1, 0, and 1. And when c=0, the $\lim_{x \to 0} f(x)$ would then equal 0 and at c= -1,1, both resulting limits would equal 1 as well.

$\lim_{x \to -1} (-1)^4 \le \lim_{x \to -1} f(x) \le \lim_{x \to -1} (-1)^2 \boxed{=1} = L$ $\lim_{x \to 0} (0)^4 \le \lim_{x \to 0} f(x) \le \lim_{x \to 0} (0)^2 \boxed{=0} = L$

$\lim_{x \to 1} (1)^4 \le \lim_{x \to 1} f(x) \le \lim_{x \to 1} (1)^2 \boxed{=1} = L$

If we were to plug in any numbers greater than 1, or -1, the resulting limits would not equal each other when the Sandwich Theorem is applied, as shown below with the following examples.

$2 > 1$ $\lim_{x \to 2} (2)^2 = 4 \ne \lim_{x \to 2} (2)^4 = 16$ $\lim_{x \to -2} (-2)^2 = 4 \ne \lim_{x \to -2} (-2)^4 = 16$

Problem # 10	Total Points 20
Understanding	4
Strategies, Reasoning, & Procedures	4
Arithmetic, Algebra, & Trigonometry	~~3~~ 4
Communication	4
Calculus Calculations	4

Problem # 66	Total Points 15
Understanding	3
Strategies, Reasoning, & Procedures	3
Arithmetic, Algebra, & Trigonometry	3
Communication	3
Calculus Calculations	3

Problem # 77	Total Points 18
Understanding	4
Strategies, Reasoning, & Procedures	4
Arithmetic, Algebra, & Trigonometry	4
Communication	2
Calculus Calculations	4