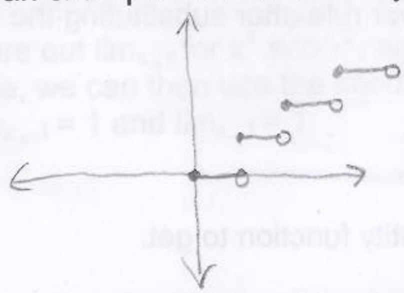
This is a student's writing assignment for 3 problems from the first assignment from Calc 1 in Fall 2011. Each question was graded on a 20 point scale (see the grading criteria elsewhere).

Because there are only a few minor problems, this assignment received a grade of 56/60.

Chapter 2.2

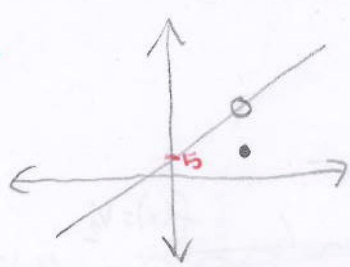
10. If f(1)=5, must $\lim_{x\to 1}$ exist? If it does, then must $\lim_{x\to 1} f(x) = 5$? Can we conclude anything about $\lim_{x\to 1} f(x)$? Explain.

This question is asking us to think about the definition of a limit and the conditions to be met in defining a limit.
If a function f(x) is equal to a certain number y at x it does not necessarily mean that the limit exists. In most cases, the limit would exist because f(x) is moving closer and closer to a specified value as the interval about x is decreasing. However, there are strange functions such as step functions that provide an exception. This is an example that came to my mind:



Alternatively, if the limit does exist when f(1)=5, then we can't necessarily say that $\lim_{x\to 1} f(x) = 5$. Again, in most occasions this would definitely be the case. But I can think of a counterexample to prove this wrong. The book says that the limit value of a function does not depend on how the function is being defined at the point being approached. Functions exist that use different equations between certain intervals that could throw this answer off. For example:

Be a little more clear w/ explanation



$$g(x) = \begin{cases} 2x+1 & x \neq 5 \\ 5 & x = 5 \end{cases}$$

We can't conclude anything with 100% certainty about $\lim_{x\to 1} f(x)$. There are strange functions that provide exceptions ~~for~~ to the rules. If there was more information provided or more stipulations added we could rule out these crazy exceptions.

About what? f(x)

66. Suppose that the inequalities

$$\frac{1}{2} - \frac{x^2}{24} < \frac{1-\cos x}{x^2} < \frac{1}{2}$$

hold for values of x close to zero. What, if anything, does this tell you about

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

This problem is asking us to use the Sandwich Theorem to make conclusions about the function in the middle. By using the sandwich theorem and limit laws we can determine the limits ~~of the two functions on either side of the inequality.~~ of the inner function.

For the first function, I used the power rule after substituting the 0 in for x and get:

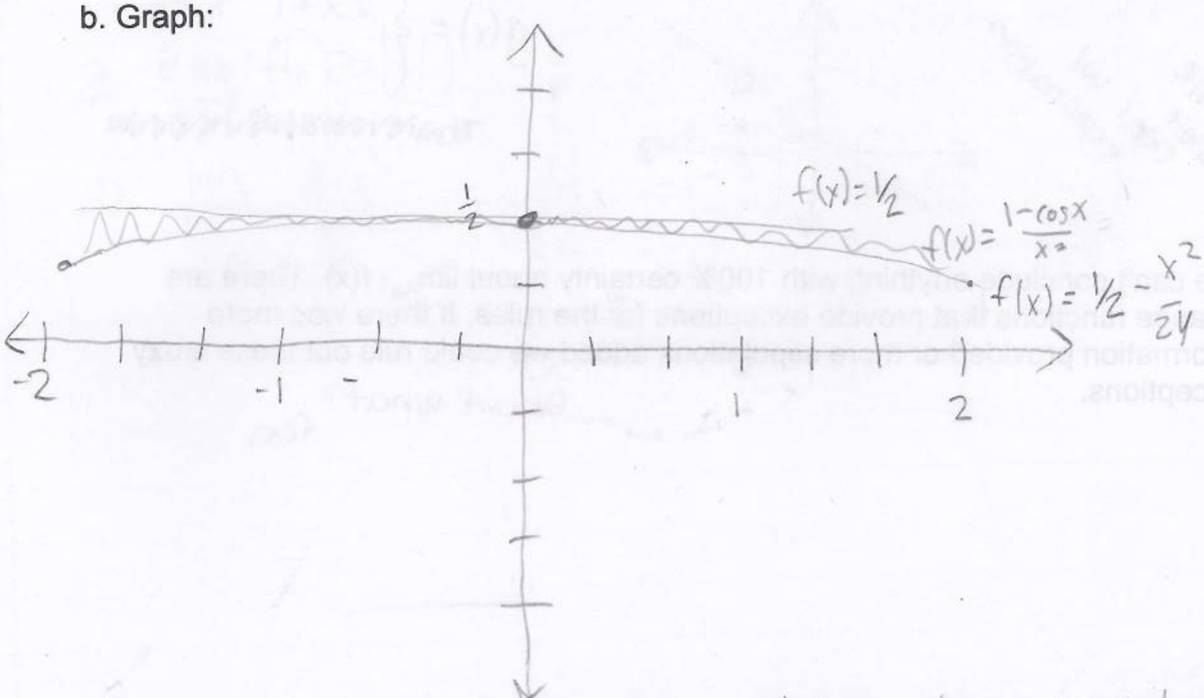
$$\lim_{x \to 0} \frac{1}{2} - \frac{(0)^2}{24} = \frac{1}{2}$$

For the last function, I used the identity function to get:

$$\lim_{x \to 0} \frac{1}{2} = \frac{1}{2}$$

Therefore, by the sandwich theorem, we can conclude that $\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

b. Graph:




all 3 graphs approach $\frac{1}{2}$ as $x \to 0$

77. If $x^4 \leq f(x) \leq x^2$ for x in [-1, 1] and $x^2 \leq f(x) \leq x^4$ for x < -1 and x > 1, at what points c do you automatically know $\lim_{x \to c} f(x)$? What can you say about the value of the limit at these points?

This problem seems a little tricky at first, but this also requires the use of the sandwich theorem. If we can figure out the limits of the functions on either side of the inequalities, we can figure out the limit of f(x). To figure this out, I needed to realize that even though the two inequalities given to us are opposite, there are still certain values that give the same answer for both $x^2$ and $x^4$, positive or negative. We can automatically know $\lim_{x \to c} f(x)$ when c = 0, 1, and -1 because these values are the same when squared or quadrupled:

$$0^2 = 0 \qquad 0^4 = 0 \qquad \lim_{x \to 0} x^4 = 0 \qquad \lim_{x \to 0} x^2 = 0$$

$$1^2 = 1 \qquad 1^4 = 1 \qquad \lim_{x \to 1} x^4 = 1 \qquad \lim_{x \to 1} x^2 = 1$$

$$(-1)^2 = 1 \qquad (-1)^4 = 1 \qquad \lim_{x \to -1} x^4 = 1 \qquad \lim_{x \to -1} x^2 = 1$$

Since we can figure out $\lim_{x \to c}$ for $x^4$ and $x^2$, and these are the same for only the values of c I chose, we can then use the sandwich theorem to conclude that $\lim_{x \to 0} f(x) = 0$, $\lim_{x \to -1} = 1$ and $\lim_{x \to 1} = 1$.