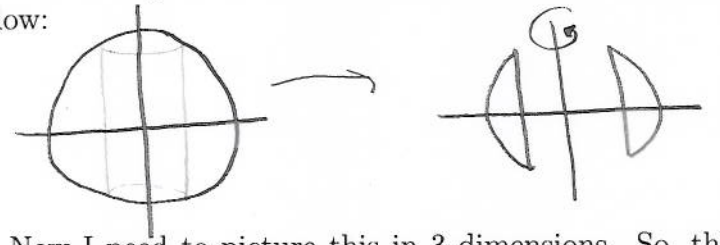
Sample Writing Assignment

On all homework assignments, you must write up your solutions clearly and in complete sentences. You should treat this like a paper. Make sure that a carbon copy of you who has not thought about this problem is able to understand your solution. Good luck!

1. A hole of radius 1 is drilled into a sphere of radius 2. Compute the volume of what remains of the sphere.

This problem involves drilling a hole through a sphere. I set this up using both the shell and washer methods. I found the shell method easier to set up but I found washer method easier to integrate.

In my mind, I could see the picture but then I had to put it onto an xy axis system. So, I looked at $x^2 + y^2 = 4$, the circle of radius 2. If a hole of radius 1 is drilled though this, I will be cutting out a vertical strip from $x = -1$ to $x = 1$ from the circle. See the picture below:

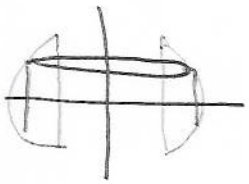


Now I need to picture this in 3 dimensions. So, think of the right side revolved about the $y-$axis. I can do this with either washers or shells. I will set up the shell method first. Because I am chopping parallel to the $y-$axis, I will be chopping up the $x-$axis and so this will be in dx. The radius is easy, it is just x. To find the height, I will take the top of the circle ($y-$value) minus the bottom of the circle (y-value) and do this in terms of x. So

$$x^2 + y^2 = 4$$

$$y^2 = 4 - x^2$$

$$y = \pm\sqrt{4 - x^2}$$



My equation will be

$$\int 2\pi x(\sqrt{4 - x^2} - (-\sqrt{4 - x^2}))\, dx$$

and the limits are in terms of x, so I go from $x = 1$ to $x = 2$. Thus

$$\int_1^2 2\pi x(2\sqrt{4 - x^2})\, dx = 4\pi \int_1^2 x\sqrt{4 - x^2}\, dx$$

I can see that this will involve a $u-$sub with $u = 4 - x^2$. I will now set it up in washers and we will see that it is easier to integrate.

My washers will be in terms of dy, and so I need the y-value where the line $x = 1$ intersects the circle. So $x^2 + y^2 = 4$ gives $1^2 + y^2 = 4$ so $y^2 = 3$ and $y = \pm\sqrt{3}$. My outside radius will be the $x-$values of the circle and my inside radius will be 1 because of the hole.

To find the outside radius, note that $x^2 = 4 - y^2$ and $x = \sqrt{4-y^2}$ (note, I can leave off the $\pm$ because I care about the positive distance. Then my set up for the washer method is:

$$\int_{-\sqrt{3}}^{\sqrt{3}} \pi(\sqrt{4-y^2})^2 - 1\, dy = 2\pi \int_{0}^{-\sqrt{3}} 4 - y^2 - 1\, dy$$

$$2\pi \int_{0}^{\sqrt{3}} 3 - y^2\, dy = 2\pi(3y - \frac{1}{3}y^3|_0^{\sqrt{3}} = 2\pi(3\sqrt{3} - \frac{3\sqrt{3}}{3}) = 4\pi\sqrt{3}$$