

1. L'Hopital's Rule

(a) Basic Problems

i. $\lim_{x \rightarrow 2} \frac{x-2}{x^2-4}$

ii. $\lim_{x \rightarrow 0} \frac{\sin(5x)}{x}$

(b) One Basic Trick

i. $\lim_{x \rightarrow 0} \frac{8x^2}{\cos(x)-1}$

ii. $\lim_{x \rightarrow \infty} \frac{\ln(x)}{2\sqrt{x}}$

iii. $\lim_{x \rightarrow \infty} \frac{e^{3x}}{x^3}$

(c) More tricky - Advanced

i. $\lim_{x \rightarrow 0} \frac{x \sin(x)}{1 - \cos(x)}$

ii. $\lim_{x \rightarrow 0^+} \sqrt{x} \ln(x)$

iii. $\lim_{x \rightarrow 0} \left(\frac{1}{\sin(x)} - \frac{1}{x} \right)$

iv. $\lim_{x \rightarrow 0^+} (1+x)^{1/x}$

v. $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + 12})$

vi. $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x} \right)^x$

2. Improper Integrals: Evaluate the following integral. Note: some of these may diverge - justify

(a) $\int_0^{\infty} e^{-x} dx$

(b) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(c) $\int_1^{\infty} \frac{1}{x^{3/2}} dx$

(d) $\int_4^{\infty} \frac{1}{x} dx$

(e) $\int_0^{\infty} xe^{-x} dx$

(f) $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$

(g) $\int_0^1 \frac{1}{x^{3/2}} dx$

(h) $\int_0^4 \frac{1}{x} dx$

(i) $\int_0^{\infty} \frac{1}{x^2 + 3x + 2} dx$