MATH 132 Worksheet 7 $n$th-term, Integral, Direct Comparison, Limit Comparison Tests

1. Evaluate the following integrals and sketch the graphs of the functions/areas:

   (a) $\int_1^\infty e^{-x}\,dx$ (b) $\int_1^\infty \frac{1}{x^2}\,dx$ (c) $\int_1^\infty \frac{1}{x}\,dx$

2. Add rectangles (above or below the curve as appropriate) to the sketches above that represent the sum of each series. Use this to determine the convergence of each series.

   (a) $\sum_{n=1}^{\infty} e^{-n}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (c) $\sum_{n=1}^{\infty} \frac{1}{n}$

3. Use the **$n$th-term test** to show that the following series diverge.

   (a) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$ (c) $\sum_{n=1}^{\infty} \frac{e^n}{\sqrt{n}}$ (e) $\sum_{n=1}^{\infty} \left(\frac{15}{4}\right)^n$

   (b) $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$ (d) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^n$

4. Determine the convergence/divergence of the following using the **integral test**.

   (a) $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$ (c) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ (e) $\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ (d) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ (f) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

5. Determine the convergence/divergence of the following using **direct comparison test**.

   (a) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ (c) $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}-1/2}$ (e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2-1/2}}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}+1}$ (d) $\sum_{n=1}^{\infty} \frac{n}{n^2-1/2}$ (f) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^5+1}}$

6. Determine the convergence/divergence of the following using the **limit comparison test**.

   (a) $\sum_{n=1}^{\infty} \frac{1}{n^2-1/2}$ (c) $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}+1/2}$ (e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1/2}}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}-1/2}$ (d) $\sum_{n=1}^{\infty} \frac{n}{n^2+1/2}$ (f) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^5-1/2}}$

7. Determine the convergence/divergence of the following. If it converges, FIND ITS SUM!

   (a) $\sum_{n=1}^{\infty} \frac{1}{2^n}$ (c) $\sum_{n=3}^{\infty} \frac{2^n}{3^{2n}}$ (e) $\sum_{n=1}^{\infty} \frac{1}{n^2+4n+3}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (d) $\sum_{n=1}^{\infty} \frac{3}{n^2+3n+2}$ (f) $\sum_{n=1}^{\infty} 3^{-n}2^{n+2}$