

1. Find the values of x for which the following series converge. Specify the *radius of convergence* and the *interval of convergence*. Justify your answers!

(a)
$$\sum_{n=1}^{\infty} x^n$$

(b)
$$\sum_{n=1}^{\infty} 3^n x^n$$

(c)
$$\sum_{n=1}^{\infty} (x-1)^{2n}$$

(d)
$$\sum_{n=1}^{\infty} 3^{n+2}(x+5)^n$$

(e)
$$\sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{7}$$

(f)
$$\sum_{n=1}^{\infty} \frac{1}{2} \left(\frac{x-3}{4} \right)^n$$

(g)
$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$$

(h)
$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n^n}$$

(i)
$$\sum_{n=1}^{\infty} \frac{n^3 x^n}{3^n}$$

(j)
$$\sum_{n=1}^{\infty} \frac{(n!)^3 x^{4n}}{(3n)!}$$

(k)
$$\sum_{n=1}^{\infty} n^2 (x-1)^n$$

2. Recall that

- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $-1 < x < 1$
- $\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1}$ for $-1 < x < 1$

Using these (and doing algebra and/or calculus), give a power series for each of the following. Make sure to indicate the interval of convergence (don't worry about the endpoints).

(a)
$$\frac{1}{1-x^2}$$

(b)
$$\frac{3x}{1+5x}$$

(c)
$$\frac{x^3}{(1+x)^2}$$

(d)
$$\frac{1}{x^2 - x^3}$$

(e)
$$\frac{2}{(1-x)^3}$$
 (Hint: take a derivative of something to get this one.)

(f)
$$-\ln|1-x|$$
 (Hint: take an integral of something to get this one.)