1. Basic: no tricks

   (a) $\int 2x^5\,dx$

   (b) $\int \frac{1}{x}\,dx$

   (c) $\int 3x^{-6}\,dx$

   (d) $\int (2e^x + 3x)\,dx$

   (e) $\int (x^{3/2} + 2x^{1/2} - 4x^{-1/2})\,dx$

2. Basic: algebraic insight

   (a) $\int \frac{1}{\sqrt{x}}\,dx$

   (b) $\int \frac{-1}{x^2}\,dx$

   (c) $\int 7(x+1)\sqrt{x}\,dx$

   (d) $\int \frac{x^3 + 4 - \sqrt{x}}{x}\,dx$

   (e) $\int (2x+1)^2\,dx$

   (f) $\int \frac{x + 2\sqrt{x-1}}{2x\sqrt{x-1}}\,dx$

3. Basic: substitution

   (a) $\int (x\sqrt{x^2+4} + 4x^7)\,dx$

   (b) $\int 3x^2(x^3+2)^7\,dx$

   (c) $\int \frac{x+1}{(x^2+2x)^7}\,dx$

   (d) $\int \sqrt{2x+7}\,dx$

   (e) $\int 2e^{2x}\,dx$

   (f) $\int \frac{x^5}{x^6+1}\,dx$

4. More complicated

   (a) $\int x\sqrt{1-x^2}\;dx$

   (b) $\int x\sqrt{1-x}\;dx$

   (c) $\int \tan x\;dx$

   (d) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\;dx$

   (e) $\int \frac{1}{x+1}\,dx$

   (f) $\int \frac{1}{x^2+1}\,dx$

   (g) $\int \frac{2x}{x^2+1}\,dx$

5. Integration by parts

   (a) $\int 3e^x\,dx$

   (b) $\int 3xe^{x^2}\,dx$

   (c) $\int 3xe^x\,dx$

   (d) $\int 3x^2e^x\,dx$

   (e) $\int x\sqrt{1-x^2}\,dx$

   (f) $\int x\sqrt{1-x}\,dx$

   (g) $\int x\sin(x)\;dx$

   (h) $\int x\sin(x^2)\,dx$

   (i) $\int x\ln(x)\,dx$

1. Integration - Trig Integrals

   (a) $\int \sin(x)\,dx$

   (b) $\int \cos(x)\sin^2(x)\,dx$

   (c) $\int \sin^2(x)\,dx$

   (d) $\int \sin^3(x)\,dx$

   (e) $\int \sin(x)\cos^4(x)\,dx$

   (f) $\int \sin^4(x)\,dx$

   (g) $\int \tan(x)\,dx$

   (h) $\int \sqrt{1-\cos(x)}\,\,dx$

   (i) $\int \tan^5(x)\sec^2(x)\,dx$

   (j) $\int \sin^2(x)\cos^2(x)\,\,dx$

   (k) $\int \sec^2(x)\,dx$

   (l) $\int \tan^2(x)\,dx$

2. Integration - Trig Sub

   (a) Identify what the trig sub should be:

      i. $\int \frac{1}{\sqrt{9+x^2}}\,dx$

      ii. $\int \frac{(1-x^2)^{5/2}}{x^8}\,dx$

      iii. $\int \frac{1}{(4-x^2)^{3/2}}\,dx$

   (b) Antidifferentiate using trig sub:

      i. $\int \frac{1}{\sqrt{9+x^2}}\,dx$

      ii. $\int (25-x^2)^{1/2}\,dx$

      iii. $\int \frac{1}{1-x^2}\,dx$

      iv. $\int \frac{1}{(4-x^2)^{1/2}}\,dx$

3. Integration - Mixed

   (a) $\int x^2\sin(x^3)\,dx$

   (b) $\int \frac{x+4}{x^2+4}\,dx$

   (c) $\int \frac{e^x}{1+e^{2x}}\,dx$

   (d) $\int (x+1)^2\sqrt{x}\,\,dx$

   (e) $\int \frac{e^x}{1+e^x}\,dx$

   (f) $\int \sin^2(x)\cos^3(x)\,dx$

   (g) $\int x\sin(x)\,dx$

   (h) $\int \tan^2(x)\sec^2(x)\,dx$

   (i) $\int x\sin^2(x)\,dx$

   (j) $\int \frac{1}{x(1+\ln(x))}\,dx$

   (k) $\int e^{\ln(x)}\,dx$

   (l) $\int \frac{x^2}{x^2+1}\,dx$

   (m) $\int ln(x)\,\,dx$

   (n) $\int e^{\sqrt{x}}\,dx$

1. Set up the partial fraction decomposition (but do not solve!)

   (a) $\dfrac{3}{x^2-4}$

   (b) $\dfrac{2}{(x^2-4)(x^2-9)}$

   (c) $\dfrac{1}{(x^2-4)(x-4)^2}$

   (d) $\dfrac{2x}{(x^2-4)(x^2+4)}$

   (e) $\dfrac{x-3}{(x^2-3x+2)(x-2)^2}$

   (f) $\dfrac{5}{x^4(x^2-1)^2(x^2+1)^3}$

2. Find the partial fraction decomposition (i.e. set them up *and* solve them!)

   (a) $\dfrac{3}{x^2-4}$

   (b) $\dfrac{x^2+4x+1}{(x-1)(x+1)(x+3)}$

   (c) $\dfrac{-2x+4}{(x^2+1)(x-1)^2}$

3. Integrate! (Use the results from above)

   (a) $\displaystyle\int \frac{3}{x^2-4}\,dx$

   (b) $\displaystyle\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx$

   (c) $\displaystyle\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$

4. Integrate!

   (a) $\displaystyle\int \frac{1}{x^2+9}\,dx$

   (b) $\displaystyle\int \sin^3(x)\,dx$

   (c) $\displaystyle\int \frac{3}{\sqrt{9-x^2}}\,dx$

   (d) $\displaystyle\int \sqrt{1-x^2}\,dx$

   (e) $\displaystyle\int \cos^2(x)\,dx$

   (f) $\displaystyle\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$

   (g) $\displaystyle\int \frac{6x+7}{(x+2)^2}\,dx$

   (h) $\displaystyle\int \frac{3x^3}{x^2-1}\,dx$

   (i) $\displaystyle\int \frac{1}{x(x^2+1)^2}\,dx$

   (j) $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,dx$

   (k) $\displaystyle\int \frac{2}{\sqrt{25-4x^2}}\,dx$

   (l) $\displaystyle\int \frac{e^x}{\sqrt{e^{2x}+9}}\,dx$

   (m) $\displaystyle\int \sin^3(x)\cos^2(x)\,dx$

   (n) $\displaystyle\int \cos(2x)\sin^9(2x)\,dx$

   (o) $\displaystyle\int 3xe^x\,dx$

MATH 132 Worksheet 4 L'Hopital's Rule and Improper Integrals

1. L'Hopital's Rule

   (a) Basic Problems

      i. $\displaystyle\lim_{x\to 2}\frac{x-2}{x^2-4}$

      ii. $\displaystyle\lim_{x\to 0}\frac{\sin(5x)}{x}$

   (b) One Basic Trick

      i. $\displaystyle\lim_{x\to 0}\frac{8x^2}{\cos(x)-1}$

      ii. $\displaystyle\lim_{x\to\infty}\frac{\ln(x)}{2\sqrt{x}}$

      iii. $\displaystyle\lim_{x\to\infty}\frac{e^{3x}}{x^3}$

   (c) More tricky - Advanced

      i. $\displaystyle\lim_{x\to 0}\frac{x\sin(x)}{1-\cos(x)}$

      ii. $\displaystyle\lim_{x\to 0^+}\sqrt{x}\ln(x)$

      iii. $\displaystyle\lim_{x\to 0}\left(\frac{1}{\sin(x)}-\frac{1}{x}\right)$

      iv. $\displaystyle\lim_{x\to 0^+}(1+x)^{1/x}$

      v. $\displaystyle\lim_{x\to\infty}(x-\sqrt{x^2+12})$

      vi. $\displaystyle\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$

2. Improper Integrals: Evaluate the following integral. Note: some of these may diverge - justify

   (a) $\displaystyle\int_0^\infty e^{-x}\,dx$

   (b) $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$

   (c) $\displaystyle\int_1^\infty \frac{1}{x^{3/2}}\,dx$

   (d) $\displaystyle\int_4^\infty \frac{1}{x}\,dx$

   (e) $\displaystyle\int_0^\infty xe^{-x}\,dx$

   (f) $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx$

   (g) $\displaystyle\int_0^1 \frac{1}{x^{3/2}}\,dx$

   (h) $\displaystyle\int_0^4 \frac{1}{x}\,dx$

   (i) $\displaystyle\int_0^\infty \frac{1}{x^2+3x+2}\,dx$

1. Determine whether the following integrals converge or diverge.

(a) $\int_1^\infty e^{-x^3}\,dx$

(b) $\int_3^\infty \frac{1}{x^{1.1}}\,dx$

(c) $\int_3^\infty \frac{1}{x^{0.9}}\,dx$

(d) $\int_3^\infty \frac{1}{\sqrt{x^2-1}}\,dx$

(e) $\int_3^\infty \frac{1}{\sqrt{x^2+1}}\,dx$

(f) $\int_5^\infty \frac{1}{(x+4)^{3/2}}\,dx$

(g) $\int_5^\infty \frac{1}{(x-4)^{3/2}}\,dx$

(h) $\int_4^\infty \frac{x}{x^3-1}\,dx$

(i) $\int_4^\infty \frac{x}{x^3+1}\,dx$

2. Evaluate the following integrals.

(a) $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

(b) $\int_0^1 \frac{1}{x^{3/2}}\,dx$

(c) $\int_0^4 \frac{1}{x}\,dx$

3. Evaluate the following limits

(a) $\lim_{x\to\infty} \frac{x^{10}}{e^x}$

(b) $\lim_{x\to\infty} \frac{(\ln(x))^2}{\sqrt{x}}$

(c) $\lim_{x\to\infty} \left(1+\frac{e}{x}\right)^x$

(d) $\lim_{x\to 0^+} \sqrt{x}\ln(x)$

(e) $\lim_{x\to 0} \frac{8x^2}{\cos(x)-1}$

(f) $\lim_{x\to\infty} \frac{\ln(x)}{2\sqrt{x}}$

(g) $\lim_{x\to\infty} \frac{e^{3x}}{x^3}$

(h) $\lim_{x\to\infty} (2x-\sqrt{4x^2+7x+12})$

4. Evaluate the following integral. Note: some of these may diverge - justify

(a) $\int \frac{x}{\sqrt{16-x^2}}\,dx$

(b) $\int \sin^3(x)\cos^4(x)\,dx$

(c) $\int \frac{1}{x(9-x^2)}\,dx$

(d) $\int \tan^4(x)\sec^2(x)\,dx$

(e) $\int_0^\infty xe^{-x}\,dx$

(f) $\int_0^\infty xe^{-x^2}\,dx$

(g) $\int x\sqrt{16-x^2}\,dx$

(h) $\int \sqrt{16-x^2}\,dx$

(i) $\int \frac{x+1}{x^2+1}\,dx$

(j) $\int x\sqrt{16-x}\,dx$

(k) $\int \frac{1}{(2x+3)^2+16}\,dx$

(l) $\int_7^\infty \frac{1}{x^2-3x-4}\,dx$

(m) $\int (x^{3/2}+x^{1/2}-\frac{4}{\sqrt{x}})\,dx$

(n) $\int (2x+1)^2\,dx$

(o) $\int \sqrt{2x+7}\,dx$

(p) $\int \frac{\sin(x)}{\cos^4(x)}\,dx$

(q) $\int \frac{x+1}{x^2+x+1}\,dx$

(r) $\int \frac{2x}{x^2+1}\,dx$

(s) $\int (x+1)^2\sqrt{x}\,dx$

(t) $\int x\sin(x)\,dx$

(u) $\int x\sin^2(x)\,dx$

(v) $\int \frac{1}{x(1+\ln(x))}\,dx$

(w) $\int \cos^3(x)\sin^2(x)\,dx$

(x) $\int \cos^2(x)\sin^2(x)\,dx$

1. Write out the first 5 terms of the sequence (beginning with $n = 1$). Do you think it will converge? To what?

   (a) $a_n = \left\{\frac{1}{n}\right\}$
   (b) $a_n = \left\{\left(\frac{1}{2}\right)^n\right\}$
   (c) $a_n = \{3^n\}$
   (d) $a_n = \left(1 - \frac{1}{n}\right)^n$
   (e) $x_n = n^{\frac{1}{n}}$
   (f) $y_n = \left(\frac{2}{5}\right)^n$
   (g) $e_n = \left(1 + \frac{2}{n}\right)^n$

2. Do the following sequences converge? Find the limit of each convergent sequence.

   (a) $b_n = \left\{\left(\frac{1}{4}\right)^n\right\}$
   (b) $a_n = \frac{\ln(n)}{n}$
   (c) $x_n = \frac{(\ln(n^2+1))^2}{n}$
   (d) $e_n = \left(1 + \frac{2}{n}\right)^n$
   (e) $p_n = \left\{\frac{2n^2 - n + 3}{n^2 + 4n + 1}\right\}$
   (f) $a_n = \frac{n + (-1)^n}{n}$
   (g) $b_n = \sqrt{\frac{2n}{n+1}}$
   (h) $g_n = \{(-1)^n\}$
   (i) $y_n = \left(1 + \frac{7}{n}\right)^n$

3. Let $S_n = \sum_{k=1}^{n} a_k$. For each of the following, find a_1, a_2, a_3, a_4 and S_1, S_2, S_3, S_4.

   (a) $a_n = \left(\frac{1}{2}\right)^n$
   (b) $a_n = \frac{1}{n}$
   (c) $a_n = \frac{1}{n^2}$
   (d) $a_n = \left(\frac{2}{3}\right)^n$

4. Do the following series converge? Find the sum of each convergent series.

   (a) $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$
   (b) $\sum_{n=6}^{\infty} \left(\frac{e}{\pi}\right)^n$
   (c) $\sum_{n=0}^{\infty} 7\left(\frac{8}{9}\right)^n$
   (d) $\sum_{n=0}^{\infty} \left(\frac{5}{2^n} - \frac{1}{3^n}\right)$
   (e) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$
   (f) $\sum_{n=1}^{\infty} (\sqrt{3})^n$
   (g) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$
   (h) $\sum_{n=1}^{\infty} e^{2n}$
   (i) $\sum_{n=1}^{\infty} e^{-2n}$

1. Evaluate the following integrals and sketch the graphs of the functions/areas:

   (a) $\displaystyle\int_1^\infty e^{-x}\,dx$ (b) $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$ (c) $\displaystyle\int_1^\infty \frac{1}{x}\,dx$

2. Add rectangles (above or below the curve as appropriate) to the sketches above that represent the sum of each series. Use this to determine the convergence of each series.

   (a) $\displaystyle\sum_{n=1}^\infty e^{-n}$ (b) $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2}$ (c) $\displaystyle\sum_{n=1}^\infty \frac{1}{n}$

3. Use the n**th-term test** to show that the following series diverge.

   (a) $\displaystyle\sum_{n=1}^\infty \frac{n}{\sqrt{n^2+1}}$ (c) $\displaystyle\sum_{n=1}^\infty \frac{e^n}{\sqrt{n}}$ (e) $\displaystyle\sum_{n=1}^\infty \left(\frac{15}{4}\right)^n$

   (b) $\displaystyle\sum_{n=1}^\infty \frac{n^2}{n^2+1}$ (d) $\displaystyle\sum_{n=1}^\infty \left(1-\frac{1}{n}\right)^n$

4. Determine the convergence/divergence of the following using the **integral test**.

   (a) $\displaystyle\sum_{n=1}^\infty \frac{1}{n^{2/3}}$ (c) $\displaystyle\sum_{n=1}^\infty \frac{n}{n^2+1}$ (e) $\displaystyle\sum_{n=3}^\infty \frac{1}{n\ln(n)}$

   (b) $\displaystyle\sum_{n=1}^\infty \frac{1}{n^{3/2}}$ (d) $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2+1}$ (f) $\displaystyle\sum_{n=1}^\infty \frac{\ln(n)}{n}$

5. Determine the convergence/divergence of the following using **direct comparison test**.

   (a) $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2+1}$ (c) $\displaystyle\sum_{n=1}^\infty \frac{1}{n^{2/3}-1/2}$ (e) $\displaystyle\sum_{n=1}^\infty \frac{1}{\sqrt{n^2-1/2}}$

   (b) $\displaystyle\sum_{n=1}^\infty \frac{1}{n^{3/2}+1}$ (d) $\displaystyle\sum_{n=1}^\infty \frac{n}{n^2-1/2}$ (f) $\displaystyle\sum_{n=1}^\infty \frac{n}{\sqrt{n^5+1}}$

6. Determine the convergence/divergence of the following using the **limit comparison test**.

   (a) $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2-1/2}$ (c) $\displaystyle\sum_{n=1}^\infty \frac{1}{n^{2/3}+1/2}$ (e) $\displaystyle\sum_{n=1}^\infty \frac{1}{\sqrt{n^2+1/2}}$

   (b) $\displaystyle\sum_{n=1}^\infty \frac{1}{n^{3/2}-1/2}$ (d) $\displaystyle\sum_{n=1}^\infty \frac{n}{n^2+1/2}$ (f) $\displaystyle\sum_{n=1}^\infty \frac{n}{\sqrt{n^5-1/2}}$

7. Determine the convergence/divergence of the following. If it converges, FIND ITS SUM!

   (a) $\displaystyle\sum_{n=1}^\infty \frac{1}{2^n}$ (c) $\displaystyle\sum_{n=3}^\infty \frac{2^n}{3^{2n}}$ (e) $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2+4n+3}$

   (b) $\displaystyle\sum_{n=1}^\infty \frac{1}{n(n+1)}$ (d) $\displaystyle\sum_{n=1}^\infty \frac{3}{n^2+3n+2}$ (f) $\displaystyle\sum_{n=1}^\infty 3^{-n}2^{n+2}$

MATH 132 Worksheet 8 Choose your own series test

Determine convergence/divergence of each series. Justify your answers!

1. $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$
2. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$
3. $\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$
4. $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}$
5. $\sum_{n=1}^{\infty} \left(1+\frac{2}{n}\right)^n$
6. $\sum_{n=1}^{\infty} \frac{2}{n^3}$
7. $\sum_{n=1}^{\infty} e^{-n}$
8. $\sum_{n=1}^{\infty} \frac{2n^2-n+3}{n^2+4n+1}$
9. $\sum_{n=1}^{\infty} \frac{n+(-1)^n}{n}$
10. $\sum_{n=1}^{\infty} (-1)^n$
11. $\sum_{n=1}^{\infty} \frac{5n^5+n^2-5n}{n^{15}-7n^{12}+8}$
12. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{3n+8}$
13. $\sum_{n=1}^{\infty} \frac{n^9-n^6+5\sqrt{n}}{n^{11}-n^5+6}$
14. $\sum_{n=1}^{\infty} \sqrt{\frac{2n}{n+1}}$
15. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$
16. $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$
17. $\sum_{n=1}^{\infty} (\sqrt{3})^n$
18. $\sum_{n=1}^{\infty} e^{-2n}$
19. $\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$
20. $\sum_{n=4}^{\infty} \frac{1}{n^2-4n+5}$
21. $\sum_{n=1}^{\infty} \ln(n)$
22. $\sum_{n=1}^{\infty} \frac{n+2}{n^3+7n-1}$
23. $\sum_{n=1}^{\infty} \frac{3+n}{5+n}$
24. $\sum_{n=1}^{\infty} ne^{-n}$
25. $\sum_{n=1}^{\infty} \frac{n+2}{n^3+7n-1}$
26. $\sum_{n=2}^{\infty} \frac{n+5}{\sqrt{n^2-1}}$
27. $\sum_{n=1}^{\infty} \frac{3}{n(n+4)}$
28. $\sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$
29. $\sum_{n=1}^{\infty} \frac{\sqrt{3n}}{(1+\sqrt{n})^5}$
30. $\sum_{n=1}^{\infty} \frac{n^{3/2}}{5n^2-1}$
31. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^8+3}}$

MATH 132 Worksheet 9 Series Tests: ratio (RaT), root (RoT), and others

1. Determine the convergence/divergence of the following series (mostly RaT):

(a) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

(b) $\sum_{n=1}^{\infty} \frac{n!}{10^n}$

(c) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

(d) $\sum_{n=1}^{\infty} \frac{5^n}{(n+1)4^{2n+1}}$

(e) $\sum_{n=1}^{\infty} ne^{-n}$

(f) $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$

(g) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^n$

(h) $\sum_{n=1}^{\infty} \frac{n^4 4^n}{n!}$

(i) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$

(j) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(k) $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

(l) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{n!}$

(m) $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$

(n) $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{(n^2)}}$

2. Determine the convergence/divergence of the following series (lots of mixed problems):

(a) $\sum_{n=1}^{\infty} \frac{n^6}{6^n}$

(b) $\sum_{n=1}^{\infty} \left(\frac{1}{n}-\frac{1}{n^2}\right)$

(c) $\sum_{n=1}^{\infty} \left(1-\frac{3}{n}\right)^n$

(d) $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!}$

(e) $\sum_{n=1}^{\infty} \frac{n!\ln(n)}{n(n+2)!}$

(f) $\sum_{n=2}^{\infty} \frac{n+5}{\sqrt{n^2-1}}$

(g) $\sum_{n=1}^{\infty} \frac{1}{(2n)!}$

(h) $\sum_{n=1}^{\infty} \frac{n^4}{n^6+3}$

(i) $\sum_{n=14}^{\infty} \frac{n6^n}{(n+1)!}$

(j) $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

(k) $\sum_{n=1}^{\infty} \left(\frac{\ln n}{\ln n^2}\right)^n$

(l) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

(m) $\sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$

(n) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^2 e^{-n}$

(o) $\sum_{n=1}^{\infty} \ln(n)$

(p) $\sum_{n=1}^{\infty} \frac{n+2}{n^3+7n-1}$

(q) $\sum_{n=1}^{\infty} \frac{16^n}{n^{120}}$

(r) $\sum_{n=1}^{\infty} \frac{3}{n(n+4)}$

(s) $\sum_{n=1}^{\infty} \frac{6+7^n}{9+7^n}$

(t) $\sum_{n=1}^{\infty} \frac{1}{6+\sqrt[4]{n^6}}$

(u) $\sum_{n=1}^{\infty} \frac{\sqrt{3n}}{(1+\sqrt{n})^5}$

(v) $\sum_{n=1}^{\infty} \frac{n^2+6n}{n^7+2}$

(w) $\sum_{n=1}^{\infty} \frac{n^{3/2}}{5n^2-1}$

(x) $\sum_{n=1}^{\infty} \left(\sqrt{n+1}-\sqrt{n}\right)$

(y) $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

(z) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+2}}\right)$

3. Determine the convergence/divergence of the following series (challenging):

(a) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ (b) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$ (c) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$ (d) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^{3/2}}$

MATH 132 Worksheet 10 (mostly) Alternating Series

A. Do the following series converge absolutely, converge conditionally, or diverge?

1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2+7}$

2. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{n+7}$

3. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n+1}{n^2+7n-1}$

4. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+2}{n^3+7n-1}$

5. $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{\ln(n)}{n^3}$

6. $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{\ln(n)}{n}$

B. Do the following series converge absolutely, converge conditionally, or diverge?

1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3+n}{5+n}$

2. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$

3. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^4+1}$

4. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{1+\sqrt{n}}$

5. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(1+\frac{1}{n}\right)^2 e^{-n}$

6. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(1-\frac{1}{n}\right)^n$

7. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$

8. $\sum_{n=1}^{\infty}(-1)^{n+1}(\sqrt{3})^n$

9. $\sum_{n=1}^{\infty}\frac{1}{(2n)!}$

10. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^4}{n^6+3}$

11. $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{n\ln(n)}$

12. $\sum_{n=1}^{\infty}\left(\frac{\ln n}{\ln n^2}\right)^n$

13. $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$

14. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+2}}\right)$

15. $\sum_{n=4}^{\infty}\frac{1}{n^2-4n+5}$

16. $\sum_{n=1}^{\infty}(-1)^{n+1}\ln(n)$

17. $\sum_{n=1}^{\infty}\frac{3}{n(n+4)}$

1. Find the values of x for which the following series converge. Specify the *radius of convergence* and the *interval of convergence*. Justify your answers!

   (a) $\sum_{n=1}^{\infty} x^n$

   (b) $\sum_{n=1}^{\infty} 3^n x^n$

   (c) $\sum_{n=1}^{\infty} (x-1)^{2n}$

   (d) $\sum_{n=1}^{\infty} 3^{n+2}(x+5)^n$

   (e) $\sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{7}$

   (f) $\sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{x-3}{4}\right)^n$

   (g) $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$

   (h) $\sum_{n=1}^{\infty} \frac{x^{2n}}{n^n}$

   (i) $\sum_{n=1}^{\infty} \frac{n^3 x^n}{3^n}$

   (j) $\sum_{n=1}^{\infty} \frac{(n!)^3 x^{4n}}{(3n)!}$

   (k) $\sum_{n=1}^{\infty} n^2 (x-1)^n$

2. Recall that

   - $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $-1 < x < 1$
   - $\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1}$ for $-1 < x < 1$

   Using these (and doing algebra and/or calculus), give a power series for each of the following. Make sure to indicate the interval of convergence (don't worry about the endpoints).

   (a) $\frac{1}{1-x^2}$

   (b) $\frac{3x}{1+5x}$

   (c) $\frac{x^3}{(1+x)^2}$

   (d) $\frac{1}{x^2-x^3}$

   (e) $\frac{2}{(1-x)^3}$ (Hint: take a derivative of something to get this one.)

   (f) $-\ln|1-x|$ (Hint: take an integral of something to get this one.)

1. Let $f(x) = \sin(x)$, $n = 5$, $c = \pi/2$, and $a = \pi$.
   (a) What is $f(a)$?
   (b) Find the Taylor polynomial of degree n (for the value of n given above) centered at $x = c$ for $f(x)$, call it $p_n(x)$.
   (c) Estimate $f(a)$ by computing $p_n(a)$.
   (d) What is the error in your estimation? (Hint: just do a simple subtraction.)
   (e) Use Taylor's Theorem to compute the bound on the error of $p_n(a)$.
   (f) Is the actual error larger or smaller than the estimate based on Taylor's theorem. Is this a stupid question? Explain.
   (g) What value of n is needed to guarantee the error is no more than 0.00001?
2. Repeat the previous question for $f(x) = \ln(x)$, $n = 7$, $c = 3$, and $a = 5$.
3. Repeat the previous question for $f(x) = e^{-x}$, $n = 6$, $c = 1$, and $a = 3$.
4. Write out the first 5 terms of the Taylor Polynomial for the following functions at the given point.
   (a) $f(x) = e^x$ about $x = 1$
   (b) $f(x) = \sqrt{x+1}$ about $x = 0$
   (c) $f(x) = \sin(x)$ about $x = 0$
   (d) $f(x) = \frac{1}{1+x}$ about $x = 0$
   (e) $f(x) = \ln(x)$ about $x = 2$
   (f) $f(x) = e^{5x^4}$ about $x = 0$
   (g) $f(x) = \ln(x + 1)$ about $x = 0$

1. Find Taylor/Maclaurin Series expansion for the following functions about the given point using the definition of Taylor series. (You may find your work on the previous worksheet helpful on some of these.)

   (a) $f(x) = e^x$ about $x = 1$

   (b) $f(x) = \sqrt{x+1}$ about $x = 0$

   (c) $f(x) = \frac{1}{1+x}$ about $x = 0$

   (d) $f(x) = \ln(x)$ about $x = 2$

   (e) $f(x) = \ln(x+1)$ about $x = 0$

   (f) $f(x) = \sin(x)$ about $x = \pi/2$

   (g) $f(x) = \cos(x)$ about $x = \pi/2$

2. Write down the Maclaurin series expansion for the following. (You should memorize these!) What is the interval of convergence for each of these?

   (a) $f(x) = \frac{1}{1-x}$ (b) $f(x) = \sin(x)$ (c) $f(x) = \cos(x)$ (d) $f(x) = e^x$

3. Using algebra/calculus and known Taylor series, find the Taylor Series expansion for the following functions about the given point.

   (a) $f(x) = \dfrac{x}{1-x}$ about $x = 0$

   (b) $f(x) = \dfrac{x^2}{1-x}$ about $x = 0$

   (c) $h(x) = \dfrac{1}{1-3x}$ centered at $x = 0$

   (d) $f(x) = e^{x^2}$ about $x = 0$

   (e) $f(x) = x\sin(x)$ about $x = 0$

   (f) $f(x) = e^{x-3}$ about $x = 0$

   (g) $f(x) = e^{5x^4}$ about $x = 0$

   (h) $f(x) = x^2\sin(3x)$ about $x = 0$

   (i) $f(x) = e^x$ about $x = 3$

   (j) $f(x) = \sin(x)$ about $x = \pi/2$

   (k) $f(x) = \cos(x)$ about $x = \pi/2$

1. Find the area bounded by the given curves.
   (a) $y = x^2$ and $y = 2x - x^2$
   (b) $y = 2x$ and $y = x^2 - 4x$
   (c) $y = 3x^2, y = x^2 + 4$
   (d) $y = \sin(x), y = \cos(x), x = 0$, and $x = \pi/2$ (Be careful!)
   (e) $y = x - 1$ and $y^2 = 2x + 6$ (Read the problem carefully!)
   (f) $y = 1/x, y = 1/x^2, x = 2$.
   (g) $y = 5\cos x, \quad y = 5 - \frac{10x}{\pi}$
   (h) $y = 36 - x^2$ and $y = x^2 - 36$
2. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.
   (a) $x + 2y = 2$, $x = 0$, $y = 0$; about the x-axis
   (b) $y = 3x^4$, $x = 1$, $x = -1$, $y = 0$; about the x-axis
   (c) $y = 3x^4$, $x = 1$, $y = 0$; about the y-axis
   (d) $y = 3x^4$, $x = 1$, $x = 0$, $y = 0$; about $x = 1$
   (e) $y = 3x^4$, $x = 1$, $x = 0$, $y = 0$; about $y = 3$
   (f) $y^2 = 4x$, $y = x$; about the x-axis
   (g) $y^2 = 4x$, $y = x$; about the y-axis
   (h) $y^2 = 4x$, $y = x$; about $x = 4$
   (i) $y^2 = 4x$, $y = x$; about $y = 4$
   (j) $x = e^{y^2}$, $y = 0$, $x = 0$, $y = 1$; about the x-axis
   (k) $y = \sin(x)$, $x = 0$, $x = \pi$, $y = 2$; about $y = 2$

1. Set up integrals (both washers and shells, if possible) that can be used to find the volume of the solid obtained by rotating the region bounded by the curves about the given line:

   (a) $y = 7x^2$, $x = 1$, and $y = 0$, about the x-axis

   (b) $f(x) = e^x$ and $y = 0$, about the x-axis over the interval $[0, 2]$

   (c) $y = 18 - x$, $y = 3x - 6$ and $x = 0$, about the y-axis

   (d) $y = x^2$ and $y = 4x$, about the line $x = 4$

   (e) $y = x^2$ and $y = 5x$, about the line $y = 0$

   (f) $x = 0$, $x = 1$, $y = 0$, and $y = 3 + x^7$, about the x-axis

   (g) $y = x^2$, $x = 3$, and $y = 0$, about the x-axis

   (h) $x = 5y$ and $y^3 = x$ (with $y \geq 0$), about the y-axis

   (i) $y = x^2$ and $y = 1$, about the line $y = 2$

   (j) $x = y^2$ and $x = 2y$, about the line $y = 2$

   (k) $x = y^2$ and $x = 1$, about the line $x = 2$

   (l) $y = x$ and $y = \sqrt{x}$, about the line $y = 1$

2. Find the length of the curves

   (a) $y = x^{1/2} - \dfrac{1}{3}x^{3/2}$ for $1 \leq x \leq 4$

   (b) $y = x^2 - (\ln(x))/8$ for $1 \leq x \leq 2$

3. Find the areas of the surfaces generated by revolving the curves about the indicated axes.

   (a) $y = x^3/9$, $0 \leq x \leq 2$, about the x-axis

   (b) $y = \sqrt{x}$, $3/4 \leq x \leq 15/4$, about the x-axis

   (c) $x = \frac{1}{3}y^{3/2} - y^{1/2}$, $1 \leq y \leq 3$, about the y-axis

1. A 5-lb bucket is lifted from the ground into the air by pulling in 20 ft of rope at a constant speed. The rope weighs 0.08 lb/ft. How much work was spent lifting the bucket and the rope?

2. How much work is performed pulling a 60 feet rope weighing 66 lbs/ft up a cliff face? At what point has half of the work been completed?

3. A cable weighing 6 lb/ft is connected to a construction elevator weighing 1500 lbs. Find the work done in lifting the elevator to a height of 500 ft.

4. A conical tank of height 20 feet and radius 8 feet, with the point at the top, is filled up to 17 feet with Dr. Pepper (63 lb/ft^3). Compute the amount of work required to pump all of the Dr. Pepper to a height of 3 feet above the top of the tank.

5. Redo the previous problem but with the tank flipped so it has the point at the bottom. Before doing it, think about which one should take more work. Do your answers support your guess?

6. A cylindrical tank of diameter 7 feet and height 13 feet is filled up to 11 feet with lead (58.9 lb/ft^3). Compute the amount of work required to pump all of the lead to a height of 5 feet above the top of the tank.

7. A hemispherical tank (with flat side up) of radius 20 ft is filled with water (weighing about 62.4 lb/ft^3) to a 15 ft depth. Find the work done in pumping all the water to the top of the tank.

8. Refer to previous problem. Find the work done in pumping all the water to a point 10 ft above the hemispherical tank.

9. A 100 lb bag of sand is lifted uniformly 120 ft in one minute. Sand leaks from the bag at a rate of 1/4 lbs/sec. What is the total work done in lifting the bag?

Solve the following differential equations. For 1, 3, 4, and 6, your answer should be in the form $y = f(x)$. For 2 and 5 you can leave the equation in another form since solving for y is probably impossible.

1. $\dfrac{dy}{dx} = \dfrac{y+1}{x}$

2. $x^2y^2dy = (y+1)dx$

3. $(e^{-y}+1)\sin(x)dx = (1+\cos(x))dy$, $y(0) = 0$

4. $x^2y' = y - yx$, $y(-1) = -1$

5. $(y - yx^2)\dfrac{dy}{dx} = (y+1)^2$

6. $x(x-2)y' + 2y = 0$, $y(3) = 6$

1. For each set of Polar coordinates (r, θ), match the equivalent Cartesian coordinates (x, y).

___1. $(4, \frac{3\pi}{2})$	A. $(0, -4)$
___2. $(-2, \frac{-2\pi}{3})$	B. $(-2\sqrt{2}, -2\sqrt{2})$
___3. $(4, \frac{-5\pi}{6})$	C. $(4\sqrt{3}, -4)$
___4. $(-8, \frac{-7\pi}{6})$	D. $(-3.5, 3.5\sqrt{3})$
___5. $(4, \frac{-3\pi}{4})$	E. $(1, 1\sqrt{3})$
___6. $(7, \frac{2\pi}{3})$	F. $(-2\sqrt{3}, -2)$

2. You are given the point $(1, \pi/2)$ in polar coordinates. (i) Find another pair of polar coordinates for this point such that $r > 0$ and $2\pi \le \theta < 4\pi$.

   $r =$ _______ $\theta =$ _______

   (ii) Find another pair of polar coordinates for this point such that $r < 0$ and $0 \le \theta < 2\pi$.

   $r =$ _______ $\theta =$ _______

3. You are given the point $(-2, \pi/4)$ in polar coordinates.

   (i) Find another pair of polar coordinates for this point such that $r > 0$ and $2\pi \le \theta < 4\pi$.

   $r =$ _______ $\theta =$ _______

   (ii) Find another pair of polar coordinates for this point such that $r < 0$ and $-2\pi \le \theta < 0$.

   $r =$ _______ $\theta =$ _______

4. Find a polar equation for the following Cartesian equations:

   (a) $x = 2$
   (b) $xy = 4$
   (c) $x^2 + y^2 = 10$
   (d) $x^2 + (y - 3)^2 = 9$
   (e) $x^2 + xy + y^2 = 1$
   (f) $y = 1$

5. Find an equation in rectangular coordinates for the following polar equations:

   (a) $r\cos(\theta) = 3$
   (b) $r^2 = 4r\cos(\theta)$
   (c) $r = \dfrac{4}{2\cos(\theta) - \sin(\theta)}$
   (d) $r = 9\sin(\theta)$
   (e) $r = 2\cos(\theta) + 2\sin(\theta)$

6. Sketch the following

   (a) $r = 1 + \cos(\theta)$
   (b) $r = 1 + 2\cos(\theta)$
   (c) $r = 4\sin(2\theta)$

7. Sketch the following and find the intersection points

   (a) $r = \sin(\theta); r = \cos(\theta)$
   (b) $r = 2; r = 2\cos(2\theta)$
   (c) $r = 2; r = 3 + 2\sin(\theta)$
   (d) $r = \sin(\theta); r = \sin(2\theta)$
   (e) $r = \cos(\theta); r = 1 - \cos(\theta)$
   (f) $r = \sin(3\theta); r = \cos(3\theta)$

1. Sketch the following and find the intersection points
   (a) $r = \sin(\theta); r = \cos(\theta)$
   (b) $r = 2; r = 2\cos(2\theta)$
   (c) $r = 2; r = 3 + 2\sin(\theta)$
2. SET UP an integral that can be used to find the area described:
   (a) Inside both of the circles: $r = \sin(\theta); r = \cos(\theta)$
   (b) Inside both of the curves: $r = 2; r = 2\cos(2\theta)$
   (c) Inside the limacon and outside the circle: $r = 2; r = 3 + 2\sin(\theta)$
3. Sketch the following and find the intersection points
   (a) $r = \sin(\theta); r = \sin(2\theta)$
   (b) $r = \cos(\theta); r = 1 - \cos(\theta)$
   (c) $r = \sin(3\theta); r = \cos(3\theta)$
4. SET UP an integral that can be used to find various areas - you choose:) You may set up more than one area per problem!
   (a) $r = \sin(\theta); r = \sin(2\theta)$
   (b) $r = \cos(\theta); r = 1 - \cos(\theta)$
   (c) $r = \sin(3\theta); r = \cos(3\theta)$
5. SET UP an integral that can be used to find the area described:
   (a) Inside the cardioid $r = 1 + \cos(\theta)$.
   (b) Inside the four leaved rose $r = 2\cos(2\theta)$.
   (c) Inside the three-petaled rose $r = 2\sin(3\theta)$.
   (d) Shared by the circles $r = 1$ and $r = 2\sin(\theta)$.
   (e) Shared by the cardioids $r = 2(1 + \cos(\theta))$ and $r = 2(1 - \cos(\theta))$.
   (f) Inside the circle $r = 3\cos(\theta)$ and outside the cardioid $r = (1 + \cos(\theta))$.