

1. Basic: no tricks

(a) $\int 2x^5 dx$

(c) $\int 3x^{-6} dx$

(d) $\int (2e^x + 3x) dx$

(b) $\int \frac{1}{x} dx$

(e) $\int (x^{3/2} + 2x^{1/2} - 4x^{-1/2}) dx$

2. Basic: algebraic insight

(a) $\int \frac{1}{\sqrt{x}} dx$

(c) $\int 7(x+1)\sqrt{x} dx$

(e) $\int (2x+1)^2 dx$

(b) $\int \frac{-1}{x^2} dx$

(d) $\int \frac{x^3 + 4 - \sqrt{x}}{x} dx$

(f) $\int \frac{x + 2\sqrt{x-1}}{2x\sqrt{x-1}} dx$ *make do this*

3. Basic: substitution

(a) $\int (x\sqrt{x^2+4} + 4x^7) dx$

(c) $\int \frac{x+1}{(x^2+2x)^7} dx$

(e) $\int 2e^{2x} dx$

(b) $\int 3x^2(x^3+2)^7 dx$

(d) $\int \sqrt{2x+7} dx$

(f) $\int \frac{x^5}{x^6+1} dx$ *do this*

4. More complicated

do these (a) $\int x\sqrt{1-x^2} dx$
(b) $\int x\sqrt{1-x} dx$

(d) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

(f) $\int \frac{1}{x^2+1} dx$

(c) $\int \tan x dx$

(e) $\int \frac{1}{x+1} dx$

(g) $\int \frac{2x}{x^2+1} dx$

5. Integration by parts

*ask
Technique
for each
of these*

(a) $\int 3e^x dx$

(d) $\int 3x^2 e^x dx$

(g) $\int x \sin(x) dx$

(b) $\int 3xe^{x^2} dx$

(e) $\int x\sqrt{1-x^2} dx$

(h) $\int x \sin(x^2) dx$

(c) $\int 3xe^x dx$

(f) $\int x\sqrt{1-x} dx$

(i) $\int x \ln(x) dx$

*do this
also for $\int \ln x dx$*

1a) $\frac{1}{3}x^6 + C$

1b) $\ln|x| + C$

1c) $-\frac{3}{5}x^{-5} + C$

1d) $2e^x + \frac{3}{2}x^2 + C$

1e) $\frac{2}{3}x^{5/2} + \frac{4}{3}x^{3/2} + 8x^{1/2} + C$

2a) $2\sqrt{x} + C$

2b) $\frac{1}{x} + C$

2c) $\frac{14}{5}x^{5/2} + \frac{14}{3}x^{3/2} + C$

2d) $\frac{1}{3}x^3 + 4\ln|x| - 2\sqrt{x} + C$

2e) $\int 4x^2 + 4x + 1 dx = \frac{4}{3}x^3 + 2x^2 + x + C$

2f) $= \int \frac{1}{2\sqrt{x-1}} + \frac{1}{x} dx = \sqrt{x-1} + \ln|x| + C$

3a) $\int x\sqrt{x^2+4} + 4x^7 dx = \int u^{1/2} \frac{1}{2} du + \frac{1}{2} x^8 = \frac{1}{3} u^{3/2} + \frac{1}{2} x^8 + C = \frac{1}{3} (x^2+4)^{3/2} + \frac{1}{2} x^8 + C$
 $u = x^2+4$
 $du = 2x dx$

3b) $\frac{1}{8}(x^3+2)^8 + C$

3c) $\frac{1}{12}(x^2+2x)^{-6} + C$

3d) $\frac{1}{3}(2x+7)^{3/2} + C$

3e) $e^{2x} + C$

3f) $\frac{1}{6} \ln|x^6+1| + C$

4a) $\int x\sqrt{1-x^2} dx = \int -\frac{1}{2} u^{1/2} du = -\frac{2}{3} u^{3/2} + C = -\frac{2}{3} (1-x^2)^{3/2} + C$
 $u = 1-x^2$
 $du = -2x dx$
 $-\frac{1}{2} du = x dx$

4b) $\int x(1-x)^{1/2} dx = \int (1-u) u^{1/2} (-du) = \int u^{3/2} - u^{1/2} du$
 $u = 1-x, du = -dx$
 $x = 1-u$
 $= \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} + C$
 $= \frac{2}{5} (1-x)^{5/2} - \frac{2}{3} (1-x)^{3/2} + C$

4c) $\int \frac{\sin x}{\cos x} dx = \ln|\cos x| + C$

4d) $2e^{\sqrt{x}} + C$

4e) $\ln|x+1| + C$

4f) $\tan^{-1} x$

4g) $\ln|x^4+1| + C$

5) a) $3e^x + C$

b) $\int 3xe^{x^2} dx = \frac{3}{2} \int e^u du = \frac{3}{2} e^u + C = \frac{3}{2} e^{x^2} + C$
 $u = x^2$
 $du = 2x dx$

c) $\int 3xe^x dx = 3xe^x - \int 3e^x dx = 3xe^x - 3e^x + C$
 $u = 3x \quad v = e^x$
 $du = 3 dx \quad dv = e^x dx$

d) $\int 3x^2 e^x dx = 3x^2 e^x - \int 6xe^x dx = 3x^2 e^x - 6xe^x + 6e^x + C$
 $u = 3x^2 \quad v = e^x$
 $du = 6x dx \quad dv = e^x dx$

e) $\int x \sqrt{1-x^2} dx = -\frac{1}{2} \int u^{1/2} du = -\frac{1}{3} u^{3/2} + C = -\frac{1}{3} (1-x^2)^{3/2} + C$
 $u = 1-x^2$
 $du = -2x dx$

f) $\int x \sqrt{1-x} dx = -\frac{2}{3} x(1-x)^{3/2} + \frac{2}{3} \int (1-x)^{3/2} dx$
 $u = x \quad v = \frac{2}{3} (1-x)^{3/2}$
 $du = dx \quad dv = -\sqrt{1-x} dx$
 $= -\frac{2}{3} x(1-x)^{3/2} - \frac{4}{15} (1-x)^{5/2} + C$

g) $\int x \sin x dx = -x \cos x + \int \cos x dx = -x \cos x + \sin x + C$
 $u = x \quad v = -\cos x$
 $du = dx \quad dv = \sin x dx$

h) $\int x \sin(x^2) dx = \int \frac{1}{2} \sin u du = -\frac{1}{2} \cos u + C = -\frac{1}{2} \cos x^2 + C$
 $u = x^2$
 $du = 2x dx$

i) $\int x \ln x dx = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$
 $u = x \quad v = \frac{1}{2} x^2$
 $du = dx \quad dv = x dx$