1. Integration - Trig Integrals

   (a) $\int \sin(x)\,dx$

   (b) $\int \cos(x)\sin^2(x)\,dx$

   (c) $\int \sin^2(x)\,dx$

   (d) $\int \sin^3(x)\,dx$

   (e) $\int \sin(x)\cos^4(x)\,dx$

   (f) $\int \sin^4(x)\,dx$

   (g) $\int \tan(x)\,dx$

   (h) $\int \sqrt{1-\cos(x)}\,dx$

   (i) $\int \tan^5(x)\sec^2(x)\,dx$

   (j) $\int \sin^2(x)\cos^2(x)\,dx$

   (k) $\int \sec^2(x)\,dx$

   (l) $\int \tan^2(x)\,dx$

2. Integration - Trig Sub

   (a) Identify what the trig sub should be:

   i. $\int \frac{1}{\sqrt{9+x^2}}\,dx$

   ii. $\int \frac{(1-x^2)^{5/2}}{x^8}\,dx$

   iii. $\int \frac{1}{(4-x^2)^{3/2}}\,dx$

   (b) Antidifferentiate using trig sub:

   i. $\int \frac{1}{\sqrt{9+x^2}}\,dx$

   ii. $\int (25-x^2)^{1/2}\,dx$

   iii. $\int \frac{1}{1-x^2}\,dx$

   iv. $\int \frac{1}{(4-x^2)^{1/2}}\,dx$

3. Integration - Mixed

   (a) $\int x^2\sin(x^3)\,dx$

   (b) $\int \frac{x+4}{x^2+4}\,dx$

   (c) $\int \frac{e^x}{1+e^{2x}}\,dx$

   (d) $\int (x+1)^2\sqrt{x}\,dx$

   (e) $\int \frac{e^x}{1+e^x}\,dx$

   (f) $\int \sin^2(x)\cos^3(x)\,dx$

   (g) $\int x\sin(x)\,dx$

   (h) $\int \tan^2(x)\sec^2(x)\,dx$

   (i) $\int x\sin^2(x)\,dx$

   (j) $\int \frac{1}{x(1+\ln(x))}\,dx$

   (k) $\int e^{\ln(x)}\,dx$

   (l) $\int \frac{x^2}{x^2+1}\,dx$

   (m) $\int ln(x)\,dx$

   (n) $\int e^{\sqrt{x}}\,dx$

(1) a) $-\cos x + C$

b) $\int \cos x \sin^2 x = \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3 x + C$
$u = \sin x,\ du = \cos x\,dx$

c) $\int \sin^2 x\,dx = \int \frac{1}{2} - \frac{\cos 2x}{2}\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

d) $\int \sin^3 x\,dx = \int (1-\cos^2 x)\sin x\,dx = -\cos x + \frac{1}{3}\cos^3 x + C$

e) $-\frac{1}{5}\cos^5 x + C$

f) $\int \sin^4 x\,dx = \int \frac{1}{4}(1-\cos 2x)^2\,dx = \frac{1}{4}\int 1 - 2\cos 2x + \cos^2 2x\,dx$

$= \frac{1}{4}(x - \sin 2x + \int \frac{1}{2}(1+\cos 4x)dx = \frac{1}{4}x - \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{32}\sin 4x + C$

g) $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\ln|\cos x| + C$

h) $\int \sqrt{1-\cos x}\,dx = \int \sqrt{2\sin^2\frac{x}{2}}\,dx = \int \sqrt{2}\sin\frac{x}{2}\,dx = -2\sqrt{2}\cos\frac{x}{2} + C$

i) $\int \tan^5 x \sec^2 x\,dx = \frac{1}{6}\tan^6 x + C$

j) $\int \sin^2 x \cos^2 x\,dx = \frac{1}{4}\int (1-\cos 2x)(1+\cos 2x)\,dx =$

$= \frac{1}{4}\int (1-\cos^2 2x)dx = \frac{1}{4}x - \frac{1}{4}\int \frac{1}{2}(1+\cos 4x)\,dx$

$= \frac{1}{4}x - \frac{1}{8}x - \frac{1}{32}\sin 4x + C = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

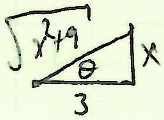
k) $\int \sec^2 x\,dx = \tan x + C$

l) $\int \tan^2 x\,dx = \int 1 - \sec^2 x\,dx = \tan x - x + C$

② (a) i) $x = 3\tan\theta$ ii) $x = \sin\theta$ iii) $x = 2\sin\theta$

(b) i) $\displaystyle\int \frac{1}{\sqrt{9+x^2}}\,dx = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9+9\tan^2\theta}} = \int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta|+C$

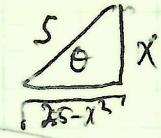
$x = 3\tan\theta$
$dx = 3\sec^2\theta\,d\theta$



$\displaystyle = \ln\left|\frac{\sqrt{x^2+9}}{3}+\frac{x}{3}\right|+C$

ii) $\displaystyle\int (25-x^2)^{1/2}\,dx = \int (25\cos^2\theta)^{1/2}\,5\cos\theta\,d\theta = \int 25\cos^2\theta\,d\theta \overset{\text{skipping details}}{=} \frac{25}{2}\theta + \frac{25}{4}\underbrace{\sin 2\theta}_{=\,2\sin\theta\cos\theta} + C$

$x = 5\sin\theta$
$dx = 5\cos\theta\,d\theta$

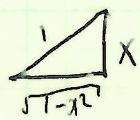


$\displaystyle = \frac{25}{2}\arcsin\left(\frac{x}{5}\right) + \frac{25}{4}\,2\,\frac{x}{5}\cdot\frac{\sqrt{25-x^2}}{5} + C$

$\displaystyle = \frac{25}{2}\arcsin\left(\frac{x}{5}\right) + \frac{1}{2}x\sqrt{25-x^2} + C$

iii) $\displaystyle\int \frac{1}{1-x^2}\,dx = \int \frac{\cos\theta\,d\theta}{\cos^2\theta\,d\theta} = \int \frac{1}{\cos\theta}\,d\theta = \int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta+C$

$x = \sin\theta$
$dx = \cos\theta\,d\theta$



$\displaystyle = \ln\left|\frac{1}{\sqrt{1-x^2}}+\frac{x}{\sqrt{1-x^2}}\right|+C = \ln\left|\frac{1+x}{\sqrt{1-x^2}}\right|+C$

$\displaystyle = \frac{1}{2}\ln\left|\left(\frac{1+x}{\sqrt{1-x^2}}\right)^2\right|+C = \frac{1}{2}\ln\left|\frac{(1+x)^2}{1-x^2}\right|+C = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|+C$

iv) $\displaystyle\int \frac{1}{(4-x^2)^{1/2}}\,dx = \arcsin\frac{x}{2}+C.$

(3)

a) $-\frac{1}{3}\cos(x^3)+C$

b) $\int \frac{x+4}{x^2+4}dx = \int \frac{x}{x^2+4}dx + \int \frac{4}{x^2+4}dx = \frac{1}{2}\int \frac{1}{u}du + \int \frac{4\cdot 2\sec^2\theta\, d\theta}{4\sec^2\theta} = \frac{1}{2}\ln|u| + 2\theta + C.$

$u = x^2+4$, $du = 2x\,dx$; $x = 2\tan\theta$, $dx = 2\sec^2\theta\, d\theta$

$= \frac{1}{2}\ln|x^2+4| + 2\arctan\frac{x}{2} + C.$

c) $\int \frac{e^x}{1+e^{2x}}dx = \int \frac{1}{1+u^2}du = \arctan u + C = \arctan e^x + C.$

$u = e^x$, $du = e^x dx$

d) $\int (x+1)^2\sqrt{x}\,dx = \int x^{5/2} + 2x^{3/2} + x^{1/2}dx = \frac{2}{7}x^{7/2} + \frac{4}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C.$

e) $\int \frac{e^x}{1+e^x}dx = \int \frac{1}{u}du = \ln|u| + C = \ln|1+e^x| + C.$

$u = 1+e^x$, $du = e^x dx$

f) $\int \sin^2 x\cos^3 x\,dx = \int \sin^2 x(1-\sin^2 x)\cos x\,dx$

$= \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$

g) $\int x\sin x\,dx = -x\cos x + \sin x + C$

$u = x$, $v = -\cos x$; $du = dx$, $dv = \sin x\,dx$

h) $\int \tan^2 x\sec^2 x\,dx = \frac{1}{3}\tan^3 x + C$

i) $\int x\sin^2 x\,dx = \frac{1}{2}\int (x - x\cos 2x)\,dx = \frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + C.$

~~$u = x$, $du = dx$~~; $u = x$, $v = \frac{1}{2}\sin 2x$; $du = dx$, $dv = \cos 2x\,dx$

j) $\int \frac{1}{x(1+\ln x)}dx = \int \frac{1}{u}du = \ln u + C = \ln|1+\ln x| + C.$

$u = 1+\ln x$, $du = \frac{1}{x}dx$

k) duh $\to \frac{1}{2}x^2 + C$

l) $\int \frac{x^2}{x^2+1}dx = \int \frac{\tan^2\theta\sec^2\theta\, d\theta}{\sec^2\theta} = \int \tan^2\theta\, d\theta$

$x = \tan\theta$, $dx = \sec^2\theta\, d\theta$

$= -\theta + \tan\theta + C$

$= -\arctan x + x + C$

m) $\int \ln x\,dx = x\ln x - \int dx = x\ln x - x + C$

$u = \ln x$, $v = x$; $du = \frac{1}{x}dx$, $dv = dx$

n) $\int e^{\sqrt{x}}dx = \int e^u 2u\,du = 2ue^u - 2e^u + C = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C.$

$u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}dx$, $2\sqrt{x}\,du = dx$; $w = 2u$, $dw = 2du$; $v = e^u$, $dv = e^u du$