

1. Do the following series converge absolutely, converge conditionally, or diverge?

$$(a) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2 + 7} \quad (c) \sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n^2 + 7n - 1} \quad (e) \sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n^3}$$

$$(b) \sum_{n=1}^{\infty} (-1)^n \frac{1}{n+7} \quad (d) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+2}{n^3 + 7n - 1} \quad (f) \sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n}$$

2. Do the following series converge absolutely, converge conditionally, or diverge?

$$(a) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3+n}{5+n}$$

$$(b) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2 + 1}$$

$$(c) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^4 + 1}$$

$$(d) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{1 + \sqrt{n}}$$

$$(e) \sum_{n=1}^{\infty} (-1)^{n+1} \left(1 + \frac{1}{n}\right)^2 e^{-n}$$

$$(f) \sum_{n=1}^{\infty} (-1)^{n+1} \left(1 - \frac{1}{n}\right)^n$$

$$(g) \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$$

$$(h) \sum_{n=1}^{\infty} (-1)^{n+1} (\sqrt{3})^n$$

$$(i) \sum_{n=1}^{\infty} \frac{1}{(2n)!}$$

$$(j) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^4}{n^6 + 3}$$

$$(k) \sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln(n)}$$

$$(l) \sum_{n=1}^{\infty} \left(\frac{\ln n}{\ln n^2}\right)^n$$

$$(m) \sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$$

$$(n) \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+2}}\right)$$

$$(o) \sum_{n=4}^{\infty} \frac{1}{n^2 - 4n + 5}$$

$$(p) \sum_{n=1}^{\infty} (-1)^{n+1} \ln(n)$$

$$(q) \sum_{n=1}^{\infty} \frac{3}{n(n+4)}$$

① a) $\sum_{n=1}^{\infty} |(-1)^n \frac{1}{n^2+7}| = \sum_{n=1}^{\infty} \frac{1}{n^2+7}$, which converges by using LCT/DCT with $1/n^2$.
Thus this series converges absolutely.

b) $\sum_{n=1}^{\infty} |(-1)^n \frac{1}{n+7}| = \sum_{n=1}^{\infty} \frac{1}{n+7}$ which diverges ($p=1$). However, $\lim_{n \rightarrow \infty} \frac{1}{n+7} = 0$,
 $\frac{1}{n+7}$ is decreasing, so $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n+7}$ converges conditionally by AST.

c) $\sum_{n=1}^{\infty} |(-1)^n \frac{n+1}{n^2+7n-1}| = \sum_{n=1}^{\infty} \frac{n+1}{n^2+7n-1}$ which diverges (use LCT with $1/n$).

but $\lim_{n \rightarrow \infty} \frac{n+1}{n^2+7n-1} = 0$, so $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n^2+7n-1}$ converges by AST. Thus it converges conditionally.

d) it converges absolutely - use LCT with $1/n^2$.

e) $\sum_{n=2}^{\infty} |(-1)^{n+1} \frac{\ln n}{n^3}| = \sum_{n=2}^{\infty} \frac{\ln n}{n^3}$ converges (see worksheet 9 problem 3b - this is the same but the sum starts at 2.)

so the series converges absolutely.

f) $\sum_{n=2}^{\infty} |(-1)^{n+1} \frac{\ln n}{n}| = \sum_{n=2}^{\infty} \frac{\ln n}{n}$ diverges by IT (see worksheet 7 problem 4f.)

however, $\lim_{n \rightarrow \infty} \frac{\ln n}{n} \stackrel{\text{LHR}}{=} \lim_{n \rightarrow \infty} \frac{1}{1} = 0$, so $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln n}{n}$ converges by AST

(note that $\ln n/n$ is eventually decreasing.). Thus the series converges conditionally.

② a) Notice that $\lim_{n \rightarrow \infty} \frac{3+n}{5+n} = 1$, so $\lim_{n \rightarrow \infty} (-1)^n \frac{3+n}{5+n}$ does not exist (it approaches 1 and -1 simultaneously). Thus the series diverges by n th term test.

b)
$$\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{n}{n^2+1} \right| = \sum_{n=1}^{\infty} \frac{n}{n^2+1} \cdot \lim_{n \rightarrow \infty} \frac{\frac{n}{n^2+1}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\frac{n^2}{n^2+1}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1}{1+\frac{1}{n^2}} = 1,$$

so this series diverges by LCT since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges ($p=1$).

However, $\lim_{n \rightarrow \infty} \frac{n}{n^2+1} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{1+\frac{1}{n^2}} = \frac{0}{1} = 0$, so the original series

converges by AST (note that $n/(n^2+1)$ decreases if $n \geq 1$).

Thus the original series converges conditionally.

c)
$$\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{n}{n^4+1} \right| = \sum_{n=1}^{\infty} \frac{n}{n^4+1}. \text{ Notice that } \frac{n}{n^4+1} < \frac{n}{n^4} = \frac{1}{n^3} \text{ by } \underline{DCT},$$

$$\sum_{n=1}^{\infty} \frac{n}{n^4+1} \text{ converges since } \sum_{n=1}^{\infty} \frac{1}{n^3} \text{ converges } (p=3>1). \text{ Therefore}$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^4+1} \text{ converges absolutely. [note that since it converges}$$

absolutely, I don't need to show that the original series converges.]

d) Use LCT to show $\sum_{n=1}^{\infty} \frac{1}{1+n}$ diverges (compare with $1/n$), and AST to show conditional convergence.

e) Use Rat to show absolute convergence. (see Worksheet 9, problem 2n.)

f) See Worksheet 9 Problem 1g to see that $\lim_{n \rightarrow \infty} (1 - \frac{1}{n})^n = \frac{1}{e}$. Thus the terms in the series alternate between adding and subtracting numbers close to $\frac{1}{e}$, so the series diverges.

- ②
- g) See worksheet 8 problem 15 to see that this series absolutely converges.
 - h) The terms of this series get larger and alternate signs, so this one diverges.
 - i) This is same as worksheet 9 problem 2g.
 - j) Use LCT with $1/n^2$ to show absolute convergence.
 - k) See worksheet 7 problem 4E to see that $\sum_{n=3}^{\infty} \frac{1}{n \ln n}$ diverges (same but series starts at 3 in that problem, doesn't change divergence.)
 but $\frac{1}{x \ln x}$ is decreasing and $\lim_{x \rightarrow \infty} \frac{1}{x \ln x} = 0$, so the series converges by AST.
 Thus it is conditionally convergent.
 - l) This is same as worksheet 9 problem 2K (converges by LCT)
 - m) See first half of 2b above - this one diverges by LCT with $1/n$.
 - n) This one converges absolutely. See worksheet 9 problem 2Z.
 - o) Converges using LCT with $1/n^2$, details omitted.
 - p) diverges. The n th term is getting larger and alternating signs, so it ^{presumably} does not converge to a single value - it gets closer to ∞ and $-\infty$.
 - q) This is same as worksheet 9 problem 2G.