1. For each set of Polar coordinates (r, θ), match the equivalent Cartesian coordinates (x, y).

___1. $(4, \frac{3\pi}{2})$	A. $(0, -4)$
___2. $(-2, \frac{-2\pi}{3})$	B. $(-2\sqrt{2}, -2\sqrt{2})$
___3. $(4, \frac{-5\pi}{6})$	C. $(4\sqrt{3}, -4)$
___4. $(-8, \frac{-7\pi}{6})$	D. $(-3.5, 3.5\sqrt{3})$
___5. $(4, \frac{-3\pi}{4})$	E. $(1, 1\sqrt{3})$
___6. $(7, \frac{2\pi}{3})$	F. $(-2\sqrt{3}, -2)$

2. You are given the point $(1, \pi/2)$ in polar coordinates. (i) Find another pair of polar coordinates for this point such that $r > 0$ and $2\pi \leq \theta < 4\pi$.

   $r =$ _______ $\theta =$ _______

   (ii) Find another pair of polar coordinates for this point such that $r < 0$ and $0 \leq \theta < 2\pi$.

   $r =$ _______ $\theta =$ _______

3. You are given the point $(-2, \pi/4)$ in polar coordinates.

   (i) Find another pair of polar coordinates for this point such that $r > 0$ and $2\pi \leq \theta < 4\pi$.

   $r =$ _______ $\theta =$ _______

   (ii) Find another pair of polar coordinates for this point such that $r < 0$ and $-2\pi \leq \theta < 0$.

   $r =$ _______ $\theta =$ _______

4. Find a polar equation for the following Cartesian equations:

   (a) $x = 2$
   (b) $xy = 4$
   (c) $x^2 + y^2 = 10$
   (d) $x^2 + (y-3)^2 = 9$
   (e) $x^2 + xy + y^2 = 1$
   (f) $y = 1$

5. Find an equation in rectangular coordinates for the following polar equations:

   (a) $r\cos(\theta) = 3$
   (b) $r^2 = 4r\cos(\theta)$
   (c) $r = \dfrac{4}{2\cos(\theta) - \sin(\theta)}$
   (d) $r = 9\sin(\theta)$
   (e) $r = 2\cos(\theta) + 2\sin(\theta)$

6. Sketch the following

   (a) $r = 1 + \cos(\theta)$
   (b) $r = 1 + 2\cos(\theta)$
   (c) $r = 4\sin(2\theta)$

7. Sketch the following and find the intersection points

   (a) $r = \sin(\theta); r = \cos(\theta)$
   (b) $r = 2; r = 2\cos(2\theta)$
   (c) $r = 2; r = 3 + 2\sin(\theta)$
   (d) $r = \sin(\theta); r = \sin(2\theta)$
   (e) $r = \cos(\theta); r = 1 - \cos(\theta)$
   (f) $r = \sin(3\theta); r = \cos(3\theta)$

① Use: $x = r\cos\theta$, $y = r\sin\theta$

e.g. 1) $x = 4\cos\frac{3\pi}{2} = 4\cdot 0 = 0$, $y = 4\sin\frac{3\pi}{2} = 4\cdot(-1) = -4$, so $(0,-4)$ $\boxed{A}$

The rest 2) $x = -2\cos(\frac{-2\pi}{3}) = -2(-.5) = 1$, $y = -2\sin(-2\pi/3) = -2(\frac{-\sqrt{3}}{2}) = \sqrt{3}$

so $(1,\sqrt{3})$ $\boxed{E}$

3) $x = 4\cos(-5\pi/6) = 4(\frac{-\sqrt{3}}{2}) = -2\sqrt{3}$, $y = 4\sin(-5\pi/6) = 4(-\frac{1}{2}) = -2$

so $(-2\sqrt{3},-2)$ $\boxed{F}$

4) $\boxed{C}$ 5) $\boxed{B}$ 6) $\boxed{D}$

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② i) $r = 1$, $\theta = 2\pi + \pi/2 = 5\pi/2$

ii) $r = -1$, $\theta = 3\pi/2$

③ i) ~~$x = -2$, $\theta = \pi/4 + 2\pi = 9\pi/2$, ii) r~~

i) $r = 2$, $\theta = \pi/4 + \pi + 2\pi = 3\pi + \pi/4 = 13\pi/4$

ii) $r = -2$, $\theta = \pi/4 - \pi = -3/4\,\pi$.

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④ a) $x = 2 = r\cos\theta$, so $\boxed{r = 2/\cos\theta}$

b) $xy = 4$, so $r\cos\theta\, r\sin\theta = 4$, $\boxed{r^2 = 4/(\cos\theta\sin\theta) = 4\sec\theta\csc\theta}$

c) $(r\cos\theta)^2 + (r\sin\theta)^2 = 10$, $r^2\cos^2\theta + r^2\sin^2\theta = r^2(\cos^2\theta + \sin^2\theta) = r^2 = 10$

or $\boxed{r = \sqrt{10}}$

d) $(r\cos\theta)^2 + (r\sin\theta - 3)^2 = 9$, $r^2\cos^2\theta + r^2\sin^2\theta - 6r\sin\theta + 9 = 9$

$r^2(\cos^2\theta + \sin^2\theta) - 6r\sin\theta = 0$, $r^2 - 6r\sin\theta = 0$ $r^2 = 6r\sin\theta$, $\boxed{r = 6\sin\theta}$

e) $r^2\cos^2\theta + r\cos\theta\, r\sin\theta + r^2\sin^2\theta = 1$, $r^2(\cos^2\theta + \sin^2\theta) + r^2\cos\theta\sin\theta = 1$

$r^2 + r^2\cos\theta\sin\theta = 1$, $r^2(1 + \cos\theta\sin\theta) = 1$, $r^2 = 1/(1 + \sin\theta\cos\theta)$

$r = \pm 1/\sqrt{1 + \sin\theta\cos\theta}$, $\boxed{r = 1/\sqrt{1 + \sin\theta\cos\theta}}$ actually suffices

f) $r\sin\theta = 1$, $r = 1/\sin\theta = \csc\theta$, so $\boxed{r = \csc\theta}$

(5) a) $r\cos\theta = 3 \Rightarrow \boxed{x=3}$

b) $r^2 = 4r\cos\theta \Rightarrow \boxed{x^2+y^2=4x}$ or $\boxed{x^2-4x+y^2=0}$

c) $r = \dfrac{4}{2\cos\theta - \sin\theta} \Rightarrow 2r\cos\theta - r\sin\theta = 4 \Rightarrow 2x - y = 4,\ \boxed{y = 2x-4}$

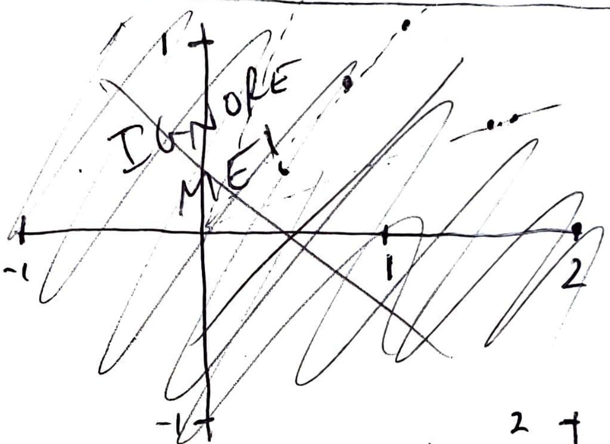
d) $r = 9\sin\theta \Rightarrow r^2 = 9r\sin\theta$ (multiplied both sides by r)

$\Rightarrow \boxed{x^2+y^2=9y}$

e) $r = 2\cos\theta + 2\sin\theta \Rightarrow r^2 = 2r\cos\theta + 2r\sin\theta \Rightarrow \boxed{x^2+y^2=2x+2y}$

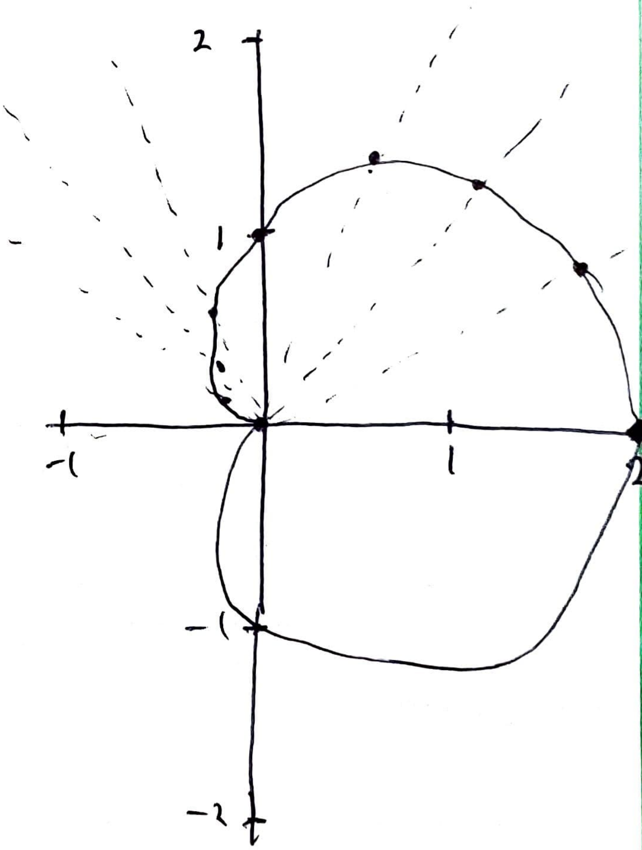
(6) a) $r = 1+\cos\theta$

θ	$1+\cos\theta$
0	2
$\pi/6$	$1+\frac{\sqrt{3}}{2} \approx 1.87$
$\pi/4$	$1+\frac{\sqrt{2}}{2} \approx 1.71$
$\pi/3$	$1+\frac{1}{2} = 1.5$
$\pi/2$	$1+0=1$
$2\pi/3$	$1-\frac{1}{2} = .5$
$3\pi/4$	$1-\sqrt{2}/2 \approx .293$
$5\pi/6$	$1-\sqrt{3}/2 \approx .134$
π	$1-1=0$



I didn't leave enough space. oops

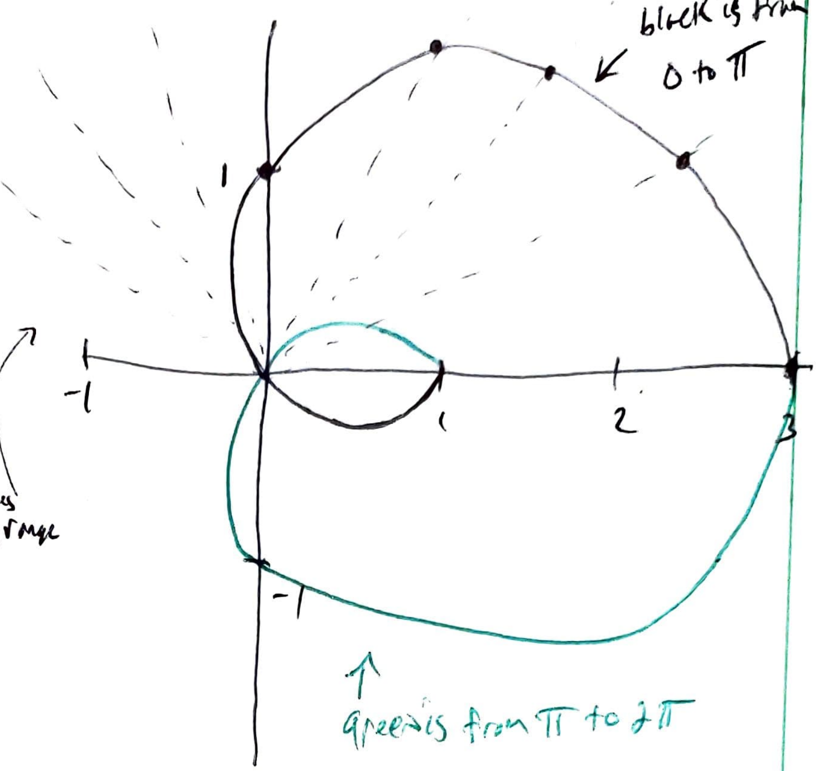
Symmetric, so I can guesstimate the bottom half.



6 b) $r = 1 + 2\cos\theta$

θ	$1+2\cos\theta$
0	$1+2=3$
$\pi/6$	$1+2\frac{\sqrt{3}}{2} \approx 2.73$
$\pi/4$	$1+2\frac{\sqrt{2}}{2} \approx 2.41$
$\pi/3$	$1+2\frac{1}{2} = 2$
$\pi/2$	$1+2\cdot 0 = 1$
$2\pi/3$	$1-2\cdot\frac{1}{2} = 0$
$3\pi/4$	$1-2\frac{\sqrt{2}}{2} \approx -.414$
$5\pi/6$	$1-2\frac{\sqrt{3}}{2} \approx -.732$
π	$1-2\cdot 1 = -1$
$\frac{7\pi}{6}$	$1+2(-\frac{\sqrt{3}}{2}) \approx -.732$

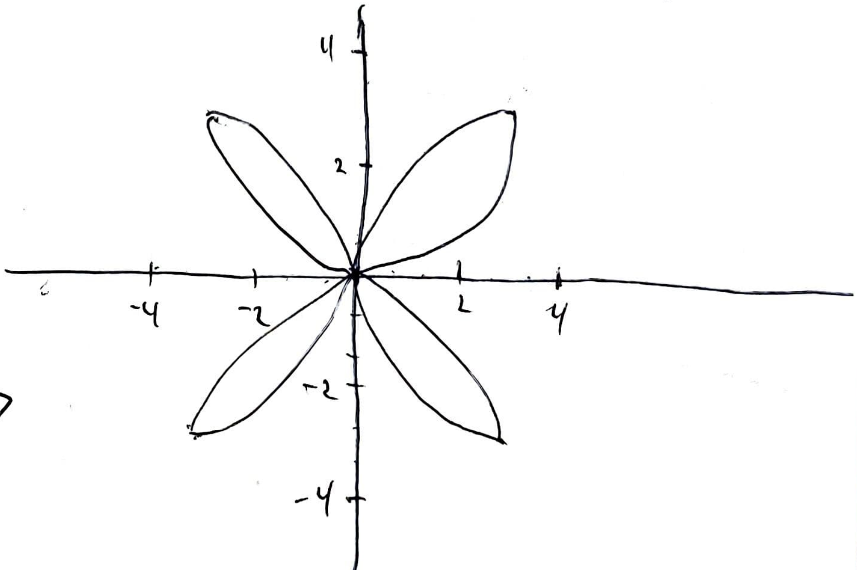
← decreases in this range



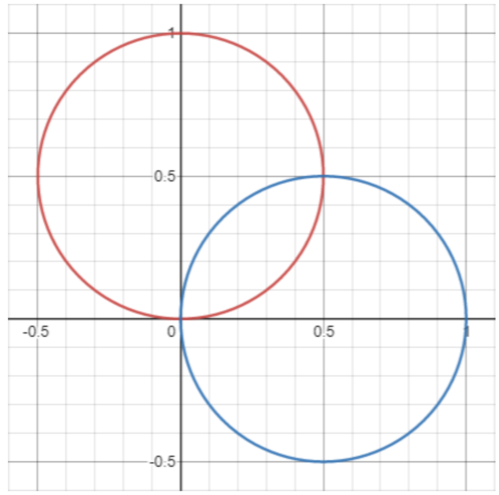
6 c) I got bored, so I made a google sheet to do the calculations, so I won't write them any more. But notice the 2θ inside the sin!

$\underline{r = 4\sin 2\theta}$ for this, smaller angles are helpful (eg $\pi/12$, etc)

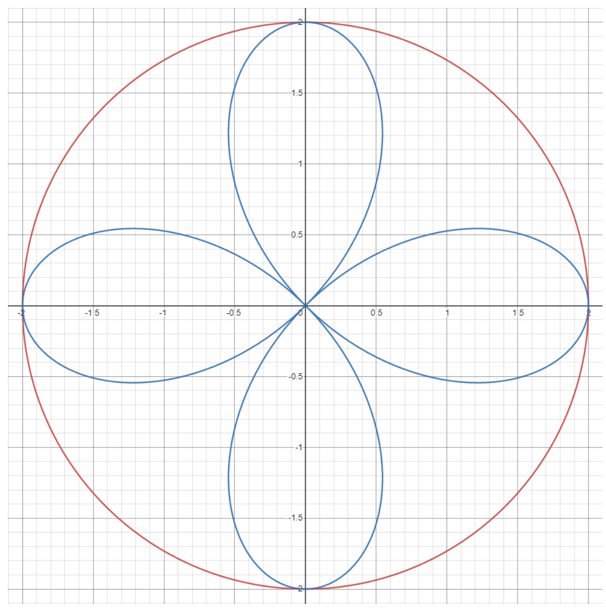
here is my not so great drawing →



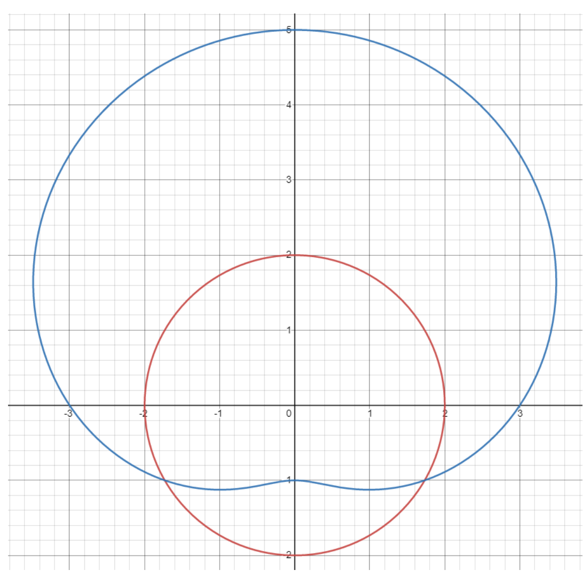
7. (a) We want $\sin\theta = \cos\theta$, which is true when $\theta = \pi/4, 5\pi/4$. Since the figures are both drawn when $0 \leq \theta \leq \pi$, we can ignore $5\pi/4$ since it is the same. Also, $r = 0$ is an intersection point. Thus they intersect at $(r,\theta) = (\sqrt{2}/2, \pi/4)$ and $r = 0$.



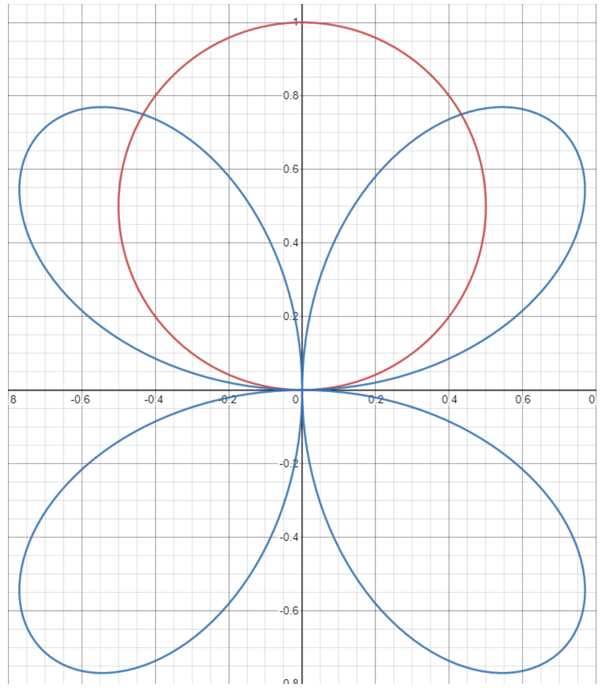
(b) They intersect when $2 = 2\cos(2\theta)$, or $\cos(2\theta) = 1$, which happens when $2\theta = 0$ and 2π, or $\theta = 0, \pi$. But this one is tricky. They also interesct when $\cos(2\theta) = -1$ (because the radius can be negative). This happens when $2\theta = \pi$ and 3π, so $\theta = \pi/2, 3\pi/2$. So the intersection points are $(2,0), (2,\pi/2), (2,\pi), (2, 3\pi/2)$.



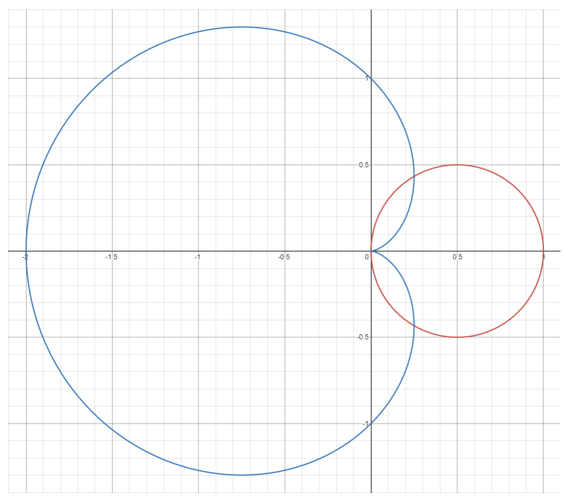
(c) Here, $2 = 3 + 2\sin(\theta)$, or $\sin(\theta) = -1/2$ which occurs when $\theta = 7\pi/6, 11\pi/6$. So the intersection points are $(2, 7\pi/6)$ and $(2, 11\pi/6)$.



(d) Here we can first use the identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$. Then we need $\sin(\theta) = 2\sin(\theta)\cos(\theta)$ or $\sin(\theta)(2\cos(\theta) - 1) = 0$. This has two solutions: when $\sin(\theta) = 0$, or when $\cos(\theta) = 1/2$. This gives $\theta = 0, \pi, 2\pi$ and $\theta = \pi/3, 5\pi/3$. The first three all give the origin as the solution (so $r = 0$). The other two give $(\sqrt{3}/2, \pi/3)$ and $(-\sqrt{3}/2, 5\pi/3)$.



(e) We have $\cos(\theta) = 1 - \cos(\theta)$, or $\cos(\theta) = 1/2$, which occurs when $\theta = \pi/3, 5\pi/3$. This gives $(1/2, \pi/3)$ and $(1/2, 5\pi/3)$. Also notice that $r = 0$ is a solution.



(f) We need $\cos(3\theta) = \sin(3\theta)$, so $3\theta = \pi/4, 5\pi/4$. This gives $\theta = \pi/12, 5\pi/12$. But we need all solutions between 0 and π (since the curve traces out between 0 and π and then repeats from π to 2π), so we also need $\theta = 9\pi/12$. This gives the points $(\sqrt{2}/2, \pi/12)$, $(\sqrt{2}/2, 5\pi/12)$, and $(\sqrt{2}/2, 9\pi/12)$. $r = 0$ is also a solution.