

Predicates and Quantifiers: Suggested Exercises

1. Write each of the following expressions so that negations are only applied to propositional functions (and not quantifiers or connectives).
 - (a) $\neg\forall x\exists y\neg P(x, y)$
 - (b) $\neg(\forall x\exists yP(x, y) \wedge \exists x\neg\forall yP(x, y))$
 - (c) $\neg\forall x(\exists yP(x, y) \vee \forall yQ(x, y))$
 - (d) $\neg\forall x\neg\exists y(\neg\forall zP(x, z) \rightarrow \exists zQ(x, y, z))$
 - (e) $\neg\exists x(\neg\forall y[\exists z(P(y, x, z) \wedge P(y, z, x) \wedge P(x, y, z))] \vee \exists zQ(x, z))$

2. Let $P(x, y)$ ="x likes y", where the universe of discourse for x and y is the set of all people. For each of the following, translate the expression to English, and tell the truth value.
 - (a) $\forall x\forall yP(x, y)$
 - (b) $\forall x\exists yP(x, y)$
 - (c) $\forall y\exists xP(x, y)$
 - (d) $\forall xP(x, \text{Chuck})$
 - (e) $\neg\forall x\forall yP(x, y)$
 - (f) $\forall x\neg\forall yP(x, y)$
 - (g) $\forall x\neg\forall y\neg P(x, y)$

3. Let $P(x, y, z)$ ="x² + y² = z²", where the universe of discourse for all variables is the set of integers. What are the truth values of each of the following?
 - (a) $\forall x\forall y\forall zP(x, y, z)$
 - (b) $\exists x\exists y\forall zP(x, y, z)$
 - (c) $\forall x\exists y\exists zP(x, y, z)$
 - (d) $\forall x\forall y\exists zP(x, y, z)$
 - (e) $\forall x\exists y\forall zP(x, y, z)$
 - (f) $\exists x\exists y\exists zP(x, y, z)$
 - (g) $\exists zP(2, 3, z)$
 - (h) $\exists x\exists yP(x, y, 5)$
 - (i) $\exists x\exists yP(x, y, 3)$

4. Write each of the following sentences using quantifiers and propositional functions (if it is possible).
 - (a) All disc golfers play ultimate frisbee.
 - (b) If all students in my class do their homework, then some of the students will pass.
 - (c) If none of the students in my class study, then all of the students in my class will fail.
 - (d) Not everybody knows how to throw a frisbee 300 feet.
 - (e) Some people like ice cream, and some people like cake, but everybody needs to drink water.
 - (f) Everybody loves somebody.
 - (g) Everybody is loved by somebody.
 - (h) Not everybody is loved by everybody.
 - (i) Nobody is loved by everybody.
 - (j) You can't please all of the people all of the time, but you can please some of the people some of the time.
 - (k) If only somebody would give me some money, I would buy a new house.
 - (l) Nobody loves me, everybody hates me, I'm going to eat some worms.
 - (m) Every rose has it's thorn, and every night has it's dawn.