# A Sample LaTeXdocument

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The purpose of this document is to serve as an example of using LaTeX. Pieces are taken from several documents, so the content will make no sense. The ideas is to give you an idea of the syntax for accomplishing various things.

Here is a *numbered list* of stuff

1. Number one
2. Number 2
3. The third.

Here is a **bulleted list**.

- The first item.
- The Second.

Somewhere near this text you should see a figure of a circuit.

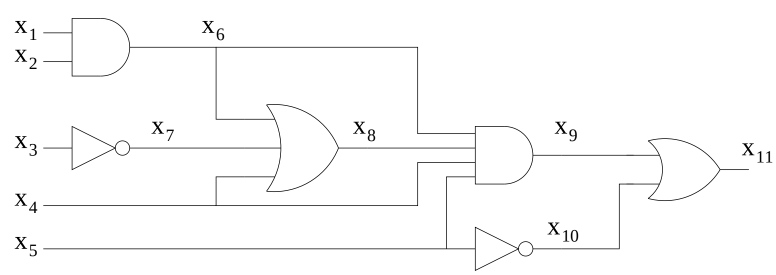


Figure 1: An instance of CIRCUIT-SAT

An example of cases:

$$c(i,j) = \begin{cases} 0, & \text{if } (i,j) \in E, \\ 1, & \text{if } (i,j) \notin E. \end{cases}$$

A Simple Table:

| Center | Right | Left | I am two columns | |
|---|---|---|---|---|
| me | Justify | justify | Blah | foo |
| | me | me | | stuff |
| WHatever I want | This doesn't mean anything | Just taking up space | | stuff |

Here is a complicated table on the same line as me.

| 1 | 2 | 3 | 4 | 5 | |
|---|---|---|---|---|---|
| 0 | | | | | 1 |
| | 0 | | | | 2 |
| | | 0 | | | 3 |
| | | | 0 | | 4 |
| | | | | 0 | 5 |

Here is an array:

| $y_2$ | $y_4$ | $y_5$ | $\varphi_{2,3}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |

A matrix: $W_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$

Some math:

$$M[1,3] = \min \left\{ \begin{array}{l} M[1,2] + M[3,3] + p_0 p_2 p_3 = 120 + 0 + 4 \cdot 3 \cdot 12 = 264 \\ M[1,1] + M[2,3] + p_0 p_1 p_3 = 0 + 360 + 4 \cdot 10 \cdot 12 = 840 \end{array} \right.$$

$$\begin{array}{rcl} x_{11} & \wedge & [(x_1 \wedge x_2) \leftrightarrow x_6] \\ & \wedge & [(\neg x_3) \leftrightarrow x_7] \\ & \wedge & [(x_9 \vee x_{10}) \leftrightarrow x_{11}] \end{array}$$

Let $x \in A \cap \overline{B}$. Then $x \in A$ and $x \in \overline{B}$ by definition of intersection ($\cap$). The skip some steps. Finally, since $A - B \subseteq A \cap \overline{B}$ and $A \cap \overline{B} \subseteq A - B$, $A - B = A \cap \overline{B}$. Also, $\lceil x + m \rceil = \lceil x \rceil + m$.

Notice that $\begin{array}{rcl} x \in A - B & \text{iff} & x \in A \text{ and } x \notin B \\ & \text{iff} & x \in A \text{ and } x \in \overline{B} \\ & \text{iff} & x \in A \cap \overline{B} \end{array}$

*Show that* $\sum_{i=1}^{n}(a_i - a_{i-1}) = a_n - a_0$, *where* $a_0$, $a_1$, ..., $a_n$ *are real numbers.*

Show that $\displaystyle\sum_{i=1}^{n}(a_i - a_{i-1}) = a_n - a_0$.

$$\begin{aligned} \sum_{i=1}^{n}(a_i - a_{i-1}) &= \left(\sum_{i=1}^{n} a_i\right) - \left(\sum_{i=1}^{n} a_{i-1}\right) \\ &= (a_1 - a_1) + (a_2 - a_2) + \cdots + (a_{n-1} - a_{n-1}) + a_n - a_0 \\ &= a_n - a_0. \end{aligned}$$

# Definitions

We say that $f(n)$ is **Big-O** of $g(n)$, written as $f(n) = O(g(n))$, iff there are positive constants $c$ and $n_0$ such that

$$0 \leq f(n) \leq c\, g(n) \text{ for all } n \geq n_0$$

If $f(n) = O(g(n))$, we say that $g(n)$ is an **upper bound** on $f(n)$.

**Theorem 1**

*Let $f(n)$ and $g(n)$ be functions such that*

$$\lim_{x\to\infty} \frac{f(n)}{g(n)} = A.$$

*Then*

1. *If $A = 0$, then $f(n) = O(g(n))$, and $f(n) \neq \Theta(g(n))$.*
2. *If $A = \infty$, then $f(n) = \Omega(g(n))$, and $f(n) \neq \Theta(g(n))$.*
3. *If $A \neq 0$ is finite, then $f(n) = \Theta(g(n))$.*

**Theorem 2: *l'Hopital's Rule***

*Let $f(x)$ and $g(x)$ be differentiable functions. If $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = 0$ or $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = \infty$, then*

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$$

# Examples

1. If $f(x) = O(g(x))$, then there are positive constants $c_2$ and $n_0'$ such that

   $$0 \leq f(n) \leq c_2\, g(n) \text{ for all } n \geq n_0'$$

   We can divide this by $c_1'$ to obtain

   $$0 \leq \frac{1}{c_1'} g(n) \leq f(n) \text{ for all } n \geq n_0''.$$

   Setting $c_1 = 1/c_1'$ and $n_0 = \max(n_0', n_0'')$, we have

   $$0 \leq c_1 g(n) \leq f(n) \leq c_2\, g(n) \text{ for all } n \geq n_0.$$

   Thus, $f(x) = \Theta(g(x))$.

2. Show that $n! = O(n^n)$
   **Proof:**
   Notice that when $n \geq 1$, $0 \leq n! = 1 \cdot 2 \cdot 3 \cdots n \leq n \cdot n \cdots n = n^n$. Therefore $n! = O(n^n)$
   (Here $n_0 = 1$, and $c = 1$.)

3. Find a tight bound on $f(x) = x^4 - 23x^3 + 12x^2 + 15x - 21$.

**Solution #1**
It is clear that when $x \geq 1$,

$$x^4 - 23x^3 + 12x^2 + 15x - 21 \leq x^4 + 12x^2 + 15x \leq x^4 + 12x^4 + 15x^4 = 28x^4.$$

Also,

$$x^4 - 23x^3 + 12x^2 + 15x - 21 \geq x^4 - 23x^3 - 21 \geq x^4 - 23x^3 - 21x^3 = x^4 - 44x^3 \geq \frac{1}{2}x^4,$$

whenever

$$\frac{1}{2}x^4 \geq 44x^3 \Leftrightarrow x \geq 88.$$

Thus

$$\frac{1}{2}x^4 \leq x^4 - 23x^3 + 12x^2 + 15x - 21 \leq 28x^4, \text{ for all } x \geq 88.$$

We have shown that $f(x) = x^4 - 23x^3 + 12x^2 + 15x - 21 = \Theta(x^4)$.

**Solution #2**
From Solution #1 we already know that $f(x) = \Theta(x^4)$. We verify this by noticing that

$$\begin{aligned} \lim_{x\to\infty} x^4 - 23x^3 + 12x^2 + 15x - 21 &= \lim_{x\to\infty} \frac{x^4}{x^4} - \frac{23x^3}{x^4} + \frac{12x^2}{x^4} + \frac{15x}{x^4} - \frac{21}{x^4} \\ &= \lim_{x\to\infty} 1 - \frac{23}{x} + \frac{12}{x^2} + \frac{15}{x^3} - \frac{21}{x^4} \\ &= \lim_{x\to\infty} 1 - 0 + 0 + 0 - 0 = 1 \end{aligned}$$

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4. Show that $(\sqrt{2})^{\log n} = O(\sqrt{n})$, where $\log$ means $\log_2$.
**Proof:**
It is not too hard to see that

$$(\sqrt{2})^{\log n} = n^{\log \sqrt{2}} = n^{\log 2^{1/2}} = n^{\frac{1}{2}\log 2} = n^{\frac{1}{2}} = \sqrt{n}.$$

Thus it is clear that $(\sqrt{2})^{\log n} = O(\sqrt{n})$.

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5. Show that $2^x = O(3^x)$.
**Proof #1:**
This is easy to see since $\lim_{x\to\infty} \frac{2^x}{3^x} = \lim_{x\to\infty} \left(\frac{2}{3}\right)^x = \lim_{x\to\infty} 0.$

**Proof #2:**
If $x \geq 1$, then clearly $(3/2)^x \geq 1$, so

$$2^x \leq 2^x \left(\frac{3}{2}\right)^x = \left(\frac{2 \times 3}{2}\right)^x = 3^x.$$